Multidimensional Array Interpolation Applied to Direction of Arrival Estimation

Marco A. M. Marinho*†, João Paulo C. L. da Costa*, Felix Antreich†, and André L. F. de Almeida‡
*Department of Electrical Engineering, University of Brasília (UnB), Brasília, Distrito Federal, Brazil
†German Aerospace Center (DLR), Institute for Communications and Navigation, Wessling, Germany
‡Department of Teleinformatics Engineering, Federal University of Ceará (UFC), Fortaleza, Brazil

***Abstract*—In MIMO communications systems, the data has a natural multidimensional structure composed of time, frequency and space dimensions. Recently, multidimensional techniques that take into account the data multidimensional structure have been proposed for model order selection, parameter estimation and prewhitening. These multidimensional techniques require an array with a PARAFAC structure. However, in practice, building antenna arrays with precise geometries is not feasible. In this paper, we propose a multidimensional array interpolation scheme that forces a real imperfect array to become a PARAFAC array. Once the multidimensional interpolation is successfully performed, advantages such as increased identifiability, separation without imposing additional constraints and improved accuracy can be exploited. Numerical simulations show that the proposed method provides improved DOA estimation accuracy when a PARAFAC technique is applied to an originally non-PARAFAC array.**

***Index Terms*—array interpolation, array mapping, multidimensional arrays, alternating least squares**

I. Introduction

Multiple-input multiple-output (MIMO) systems are already incorporated into 4G communication systems in order to increase their transmission rates and also the link reliability. Although the amount of antennas is small in current commercial systems, in future systems, arrays with several antennas such as planar arrays are expected. Multidimensional antenna arrays allow the estimation of spatial parameters such as the azimuth and the elevation of multipath components of the MIMO channel. The increase in the number of dimensions is very beneficial for signal processing, offering improved spatial resolution and identifiability. However, it also leads to some design challenges. Antennas rarely have symmetrical and identical patterns, and while it is easier to guarantee a certain degree of uniformity over a single dimension, it becomes increasingly challenging to offer such precise chracteristics over multiple dimensions.

Important direction of arrival (DOA) estimation techniques such as Iterative Quadratic Maximum Likelihood (IQML) [1], Root-WSF [2] and Root-MUSIC [3] all rely on a Vandermonde or centro-hermitian array response. Spatial Smoothing (SPS) [4] and Forward Backward Averaging (FBA) [5] are two very important array signal processing techniques that can improve the accuracy of parameter estimation in the presence of correlated or coherent signals. However, they also require an array with a Vandermonde and centro-hermitian response, respectively. Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [6] is a precise and fast DOA estimation approach that demands a shift invariant array response. In the case of multidimensional arrays, usually the parallel factor analysis (PARAFAC) structure is assumed, which means that the data tensor can be decomposed into a sum of rank-one tensors, where each tensor dimension is coupled to the other via outer products [7], [8]. Once the PARAFAC structure is present, the multidimensional problem can be easily decomposed into several one dimensional problems reducing significantly the problem complexity [8].

Arrays with perfect Vandermonde or centro-hermitian or even PARAFAC responses are very hard to obtain in practice, since antenna elements often do not present a flat response with respect to the azimuth and elevation of the received signal. Each antenna element has a unique response and the antennas interfere with each due to the mutual coupling. To help mitigate the effects of imperfect array responses the process of transforming real imperfect array responses into desired ideal array responses by means of a transformation matrix, also known as array interpolation, was first proposed in [9].

Most array interpolation schemes divide the complete angular region into limited angular sectors. For each sector a mapping/transformation matrix is defined using knowledge of the empirical measured array response. Then after transformation to a desired ideal virtual array, FBA, SPS [10] and DOA estimation algorithms such as Root-MUSIC [11] can be applied. However, when performing array interpolation with a sector-by-sector processing the mapping matrices have to be carefully derived in order to minimize the transformation bias within each sector and on the other hand to control its out-of-sector response. The out-of-sector response was neglected in earlier works [10], [11], [12], [13]. Addressing the out-of-sector response by a signal adaptive weighting and a sector-by-sector estimation of highly correlated and closely spaced signal environments is proposed in [14] and [15]. Furthermore, although before the array interpolation the noise is white, after the array interpolation the noise becomes colored. Therefore, a prewhitening step is necessary for MUSIC [16] and Root-MUSIC algorithms [3],[11]. Array interpolation techniques that allow the application of a modified ESPRIT algorithm have been proposed in [17] and [18]. These techniques do not require the prewhitening step, thus allowing the direct application of the ESPRIT algorithm. However, they ignore the out-of-sector response and they do not consider the application of FBA or/and SPS and thus cannot be applied in the case of correlated signals. In this work a signal adaptive approach is developed for interpolating multidimensional arrays with non-PARAFAC structures into arrays with the PARAFAC structure. The work from [19] is chosen as the basis to be extended for the calculation of the transformation matrices since it is signal adaptive and capable of focusing the transformation into areas where significant power is received. Moreover, the scheme in [19] outperforms the state-of-the-art scheme in the literature.

To the best of our knowledge, all array interpolation approaches in the literature only take into account unidimensional arrays. Extending the unidimensional interpolation approach to the multidimensional case is usually a straightforward process. However, the problem of discretizing and constructing the interpolation sectors becomes more complex due to the coupling between dimensions in the array response. With the advent of high-dimensional data, having dimensions such as frequency or polarization, directly applying classic array interpolation techniques may become impractical. High dimensional data with specific structures such as the parallel factor analysis (PARAFAC) data model are of special interest.

In this paper, we propose a multidimensional interpolation scheme that transforms an imperfect array into an ideal PARAFAC array. Once the multidimensional interpolation is successfully applied, the advantages of multidimensional array signal processing schemes can be guaranteed. The first advantage is the increased identifiability, which means in a telecommunication context that more subscribers can be served. Consequently, a tensor based system is more efficient in terms of resource usage than a matrix based system. A second advantage of tensors is that multilinear mixtures can be unmixed uniquely without imposing additional constraints. For matrices, i.e. bilinear problems, this requires posing additional artificial constraints, such as orthogonality leading to Principal Component Analysis (PCA) or statistical independence leading to Independent Component Analysis (ICA), which may not be physically reasonable. Finally, the third advantage is the 'tensor gain' in terms of an improved accuracy, since we can reject the noise more efficiently, filtering out everything that does not fit to the structure [20]. In addition, once the multidimensional interpolation is successfully performed, multidimensional schemes for model order selection [21], parameter estimation [8], [22] and prewhitening [23], [24] can be applied.

The remainder of this work is divided as follows. Section II presents a brief overview on tensor calculus. In Section III the data model is proposed. Section IV presents the proposed multidimensional interpolation approach. Section V presents a set of numerical simulation and discusses its results. Finally, conclusions are drawn in Section VI.

II. Tensor Calculus

The r-th unfolding of a tensor $\mathbf{A}$, $[\mathbf{A}]_{(r)}$, is a matrix containing the r-mode vectors or r fibers of tensor $\mathcal{A}$ along its rows. An r-mode vector can be obtained by fixing the index of all dimensions other than r and varying the index of the r-th dimension along its range.

The r-mode product of a tensor $\mathcal{A} \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times M_r}$ and a matrix $\mathbf{D} \in \mathbb{C}^{L \times M_r}$ is denote by

$$\mathcal{C} = \mathcal{A} \times_r \mathbf{D} \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times L \times \ldots \times M_r}, \quad (1)$$

which in the matrix form is given by

$$[\mathbf{C}]_{(r)} = \mathbf{D}[\mathbf{A}]_{(r)}. \quad (2)$$

III. Data Model

We consider d planar wave fronts are impinging on an antenna array containing $M_1 \times M_2 \times ... \times M_R$ elements. The signal received at the $m_1, m_2, \ldots m_R$ antenna at a given time snapshot t is given by

$$x_{m_1,m_2,\ldots m_R,t} = \sum_{i=1}^{d} \gamma_{m_1,m_2,\ldots m_R,d} s_i(t) \prod_{r=1}^{R} e^{j\cdot(m_r-1)\cdot\mu_i^{(r)}} + n_{m_1,m_2,\ldots m_R,t}, \quad (3)$$

where $s_i(t)$ is the complex symbol transmitted by the i-th source at time snapshot t, $n_{m_1,m_2,\ldots m_R,t}$ is the zero mean circularly symmetric (ZMCS) white complex Gaussian noise present at antenna $m_1, m_2, \ldots m_R$ at time snapshot t. $\gamma_{m_1,m_2,\ldots m_R}$ is an imperfection present at the antenna $m_1, m_2, \ldots m_R$. $\mu_i^{(r)}$ represents the spatial frequency of the signal transmitted by the i-th source over the r-th dimension. Note that the spatial frequency can be mapped into different parameters depending on the dimension. In case of spatial dimension, we map it to direction of arrival, while for frequency and time dimensions we can map it to time delay of arrival and Doppler shift, respectively.

For the received signal to have the PARAFAC structure its steering array must have a Khatri-Rao structure as follows. Let M_r be the size of the r-th dimension, and $M = \prod_{r=1}^{R} M_r$. A steering vector $\mathbf{a}_i^{(r)}$ containing the spatial frequencies relate to the i-th source over the r-th dimension can be defined as

$$\mathbf{a}_i^{(r)} = \begin{bmatrix} 1 & e^{j\cdot\mu_i^{(r)}} & \cdots & e^{(M_r-1)\cdot j\cdot\mu_i^{(r)}} \end{bmatrix}^{\mathrm{T}}. \quad (4)$$

Therefore, an array steering vector for the i-th source can be written as

$$\mathbf{a}_i = \mathbf{a}_i^{(1)} \otimes \mathbf{a}_i^{(2)} \otimes \ldots \otimes \mathbf{a}_i^{(r)} \in \mathbb{C}^{M \times 1}, \quad (5)$$

while the array steering matrix for the r-th dimension can be constructed as

$$\mathbf{A}^{(r)} = \left[\mathbf{a}_1^{(r)}, \mathbf{a}_2^{(r)}, \ldots, \mathbf{a}_d^{(r)}\right] \in \mathbb{C}^{M \times d}. \quad (6)$$

Finally, the total steering matrix of the d sources can be constructed in terms of Khatri-Rao products of all array steering matrices of all dimensions.

$$\mathbf{A} = \mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)} \diamond \ldots \diamond \mathbf{A}^{(r)} \in \mathbb{C}^{M \times d}. \quad (7)$$

For a steering array with a non steering Khatri-Rao structure the received signal matrix is given by

$$\mathbf{X} = (\mathbf{A} \odot \mathbf{\Gamma}) \cdot \mathbf{S} + \mathbf{N} \in \mathbb{C}^{M \times N}, \quad (8)$$

where $\odot$ is the Hadamard product, the symbol matrix $\mathbf{S} \in \mathbb{C}^{d \times N}$, where N is the number of times snapshots taken, contains the symbols $s_i(t)$ transmitted by the d sources. The matrix $\mathbf{N} \in \mathbb{C}^{M \times N}$ contains the white Gaussian noise samples. Finally, $\mathbf{\Gamma} \in \mathbb{C}^{M \times d}$ is a matrix containing the antenna imperfections. The resulting $\mathbf{X} \in \mathbb{C}^{M \times N}$ matrix contains the measurements of one snapshot stacked along one column, with the snapshots taken along different dimensions stacked along its rows.

For the tensor notation, a PARAFAC steering tensor for each of the d signal sources can be obtained by

$$\mathcal{A}_i = \mathbf{a}_i^{(1)} \circ \mathbf{a}_i^{(2)} \circ \ldots \circ \mathbf{a}_i^{(r)} \in \mathbb{C}^{M_1 \times M_2 \times \cdots \times M_R}, \quad (9)$$

where $\circ$ is the outer product. A total array steering tensor can be constructed by concatenating the individual steering tensor for each of the signal sources over the $r+1$ dimension, as it follows

$$\mathcal{A} = [\mathcal{A}_1 |_{R+1} \mathcal{A}_2 |_{R+1} \ldots |_{R+1} \mathcal{A}_d] \in \mathbb{C}^{M_1 \times M_2 \times \cdots \times M_R \times d}. \quad (10)$$

The $|_r$ operator represents the concatenation operation over the r-th dimension.

A non PARAFAC steering tensor can be written as

$$\mathcal{X} = (\mathcal{A} \odot \mathcal{G}) \times_R \mathbf{S}^{\mathrm{T}} + \mathcal{N}, \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times M_R \times N}, \quad (11)$$

here $\mathcal{N} \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times M_R \times N}$ is the noise tensor and contains the noise samples interfering with the measurements, and $\mathcal{G} \in \mathbb{C}^{M_1 \times M_2 \times \cdots \times M_R \times d}$ is a tensor containing the antenna imperfections.

IV. Multidimensional Array Interpolation

Array interpolation aims to predict what signal would be received at an antenna array with a specific desired geometry based on the signal that was received by a real antenna array. In matrix form the transformation tries to achieve

$$\mathbf{B}\mathbf{A}_{\mathcal{S}} = \bar{\mathbf{A}}_{\mathcal{S}}, \quad (12)$$

where $\mathbf{A}_{\mathcal{S}}$ and $\bar{\mathbf{A}}_{\mathcal{S}}$ are array response matrices constructed considering the discrete set of angles

$$\mathcal{S} = \{l_{\mathcal{S}}, l_{\mathcal{S}} + \Delta, ..., u_{\mathcal{S}} - \Delta, u_{\mathcal{S}}\}. \quad (13)$$

Here, $l_{\mathcal{S}}$ is the lower bound, $u_{\mathcal{S}}$ is the upper bound of sector $\mathcal{S}$ and Δ is the angular resolution of the transformation. The matrix $\mathbf{B}$ can be seen as the matrix that achieves the best transform between a set of vectors $\mathbf{A}_{\mathcal{S}}$ and $\bar{\mathbf{A}}_{\mathcal{S}}$. The transformation matrix $\mathbf{B}$, however, is usually not capable of transforming the response perfectly across the entire sector $\mathcal{S}$ except for the case where a large number of antenna elements is present or a very small sector is used. The error of the transform is defined as

$$\epsilon(\mathcal{S}) = \frac{\left\|\bar{\mathbf{A}}_{\mathcal{S}} - \mathbf{B}\mathbf{A}_{\mathcal{S}}\right\|_{\mathrm{F}}}{\left\|\bar{\mathbf{A}}_{\mathcal{S}}\right\|_{\mathrm{F}}} \in \mathbb{R}^{+}. \quad (14)$$

Larger sectors will lead to larger transformation errors, and while it is possible to keep the transformation error as low as desired by keeping the sector sizes small this may lead to further problems such as demanding a very large number of estimations to be performed, one for each sector. By increasing the number of antennas at the virtual array to obtain a smaller transformation error, the transformation matrix becomes ill-conditioned causing a large bias in the final DOA estimates. Therefore, the number of antennas in the virtual array is usually chosen to be equal or smaller then the number of antennas in the real array.

In the multidimensional case we have

$$\mathcal{I} \times_1 \bar{\mathbf{A}}_1 \times_2 \bar{\mathbf{A}}_2 ... \times_R \bar{\mathbf{A}}_R = \mathcal{A} \times_1 \mathbf{B}_1 \times_2 \mathbf{B}_2 ... \times_R \mathbf{B}_R, \quad (15)$$

where $\bar{\mathbf{A}}_r$ is the desired array response for the r-th dimension and $\mathbf{B}_r$ is the transformation matrix for the r-th dimension given by

$$\mathbf{B}_r = [\mathcal{A}]_{(r)} \bar{\mathbf{A}}_r^{\dagger}, \quad (16)$$

where $(\boldsymbol{.})^{\dagger}$ stands for the Moore–Penrose pseudo-inverse. Note that in (15) the original array $\mathcal{A}$ has no PARAFAC structure, while the array after the interpolation has the PARAFAC structure.

A. Multidimensional Sectoring

The first step in order to interpolate the array response is to detect the regions of the manifold where the signals are received in order to decide what regions of the manifold must be interpolated, extending the work from [19]. The power response for dimension r is given by

$$P(\mu^{(r)}) = \left| \frac{\mathbf{w}^{(r)\mathrm{H}}(\mu^{(r)}) \hat{\mathbf{R}}_{\mathbf{XX}}^{(r)} \mathbf{w}^{(r)}(\mu^{(r)})}{\mathbf{w}^{(r)\mathrm{H}}(\mu^{(r)}) \mathbf{w}^{(r)}(\mu^{(r)})} \right| \in \mathbb{R}, \quad (17)$$

where $(\cdot)^{\mathrm{H}}$ is the conjugate hermitian and $\hat{\mathbf{R}}_{\mathbf{XX}}^{(r)}$ is the spatial covariance matrix for dimension (r). $\hat{\mathbf{R}}_{\mathbf{XX}}^{(r)}$ is given by

$$\hat{\mathbf{R}}_{\mathbf{XX}}^{(r)} = \frac{M_r}{N \prod_{i=1}^{R} M_i} [\mathcal{X}]_{(r)} [\mathcal{X}]_{(r)}^{\mathrm{H}} \in \mathbb{C}^{M_r \times M_r}. \quad (18)$$

In real systems the result of (17) is discrete in $\mu^{(r)}$ and can be written as

$$P[z]^{(r)} = P(-\frac{\pi}{2} + (z \cdot \Delta)) = P(\mu^{(r)}), \quad (19)$$

with $\mu^{(r)} \in \mathcal{D}_{\Delta}$ where

$$\mathcal{D}_{\Delta} = \{-\frac{\pi}{2}, -\frac{\pi}{2} + \Delta, ..., \frac{\pi}{2} - \Delta, \frac{\pi}{2}\}, \quad (20)$$

and Δ is the resolution of the power response (17).

The output of (17) is scanned for K sectors, and for each sector the respective lower bound $l_k^{(r)} \in \mathcal{D}_{\Delta}$ and upper bound $u_k^{(r)} \in \mathcal{D}_{\Delta}$, where $k = 1, \ldots, K$, are defined as shown in Figure 1.

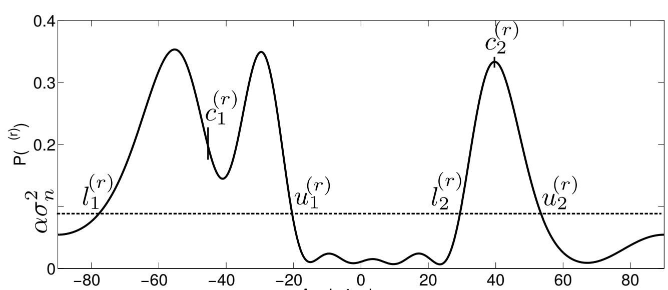

Figure 1: Example of detected regions

The threshold that defines a sector and its bounds can be defined, for instance, considering the noise power. The noise floor can be set at $\alpha\sigma_n^2$, with $\alpha > 1$ being a sensitivity parameter and σ_n^2 being the noise variance. A large α means that a sector is only detected in case of large signal power but coming at the cost of discarding regions where the received signal power is small. A small α means regions with smaller detected power are considered to be a sector, but at the cost of allowing regions with only noise to be mistakenly be considered as a sector. If K sectors are detected, a detected sector with bounds $[l_k^{(r)}, u_k^{(r)}]$ is said to be centered at

$$c_k = \left\lceil \frac{\left| u_k^{(r)} - l_k^{(r)} \right|}{2} \right\rfloor_{\mathcal{D}_\Delta} \in \mathcal{D}_\Delta, \tag{21}$$

where $\lceil \cdot \rfloor_{\mathcal{D}_\Delta}$ is a rounding operation to the domain $\mathcal{D}_\Delta$. A weighting factor for each sector is calculated as

$$\xi_k^{(r)} = \frac{\sum_{z=l_k^{(r)}}^{u_k^{(r)}} P[z]^{(r)}}{\sum_{w=1}^{K} \sum_{z=l_w^{(r)}}^{u_w^{(r)}} P[z]^{(r)}} \in \mathbb{R}. \tag{22}$$

Classical array interpolation methods in the literature divide the array response into various sectors (sector-by-sector processing) in order to keep the error (14) small. However, in this work a single transformation matrix for each dimension is used based on a combination of the sectors detected in (17). In order to bound the error $\epsilon(\mathcal{S}^{(r)})$ a signal adaptive method for calculating the maximum transform size is used based on the weights calculated in (22). For a sector centered at $c^{(r)}{}_k$, the discrete and countable set of angles used to transform this sector is given by (24) and

$$\mathcal{S}_k^{(r)} \cap \mathcal{S}_{\bar{k}}^{(r)} = \emptyset \; \forall \; k, \bar{k} \in \{1, ..., K\} \text{ and } k \neq \bar{k}, \tag{23}$$

where $\Xi \in \mathbb{R}^+$ is the total transformation size in radians of all sectors. The combined sector for the r-th dimension is given by

$$\mathcal{S}^{(r)} = \mathcal{S}_1^{(r)} \cup \mathcal{S}_2^{(r)} \cup ... \cup \mathcal{S}_K^{(r)}. \tag{25}$$

Thus, $\mathcal{S}^{(r)}$ has a "wider" support for the sectors $\mathcal{S}_k^{(r)}$ where more power is present (weighted by $\xi_k^{(r)}$), i.e the transformation of the combined sector $\mathcal{S}^{(r)}$ is weighted towards the sectors $\mathcal{S}_k^{(r)}$ that include more signal power.

As the problem of obtaining the transform matrix $\mathbf{B}_r$ is equivalent to solving a highly overdetermined system we have

$$|\mathcal{S}^{(r)}| \to \infty \Longleftrightarrow \epsilon(\mathcal{S}^{(r)}) \to \infty. \tag{26}$$

Thus, transforming the entire detected sectors may result in a very high transformation error introducing a very large bias into the final DOA estimates. To address this problem an upper bound to the transformation error $\epsilon_{\max}$ needs to be defined and a search can be performed to find the maximum transformation size covering the detected sectors that is still within the error upper bound. The problem of finding the maximum Ξ with respect to $\epsilon_{\max}$ can be written as the optimization problem

$$\max_{\Xi} \epsilon(\mathcal{S}^{(r)}) \tag{27}$$

$$\text{subject to } \epsilon(\mathcal{S}^{(r)}) \leq \epsilon_{\max} \tag{28}$$

$$\Xi \leq \Xi_{\max} = \sum_{k=1}^{K} |u_k^{(r)} - l_k^{(r)}| \tag{29}$$

$$\Xi \geq \Xi_{\min} = M_r \Delta. \tag{30}$$

The problem in (27)-(30) can efficiently be solved using a bisection search method since, once all $c_k^{(r)}$ have been defined, the error function increases monotonically for $\Xi > \Xi_{\min}$, as illustrated in Figure 2. $\epsilon(\mathcal{S}^{(r)})$ is greatly affected if the calculation of $\mathbf{B}_r$ is either a heavily overdetermined or an underdetermined system. Therefore, $\Xi_{\min}$ is defined to ensure monotonicity of the problem given in (27).

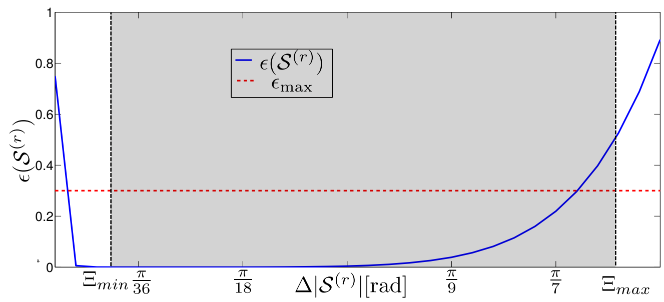

Figure 2: Transformation error with respect to combined sector size

Alternatively, the instead of solving the optimization problem the sectors can be discretized using the unscented transform as shown in [25].

B. Transformation Matrix Calculation

Once the regions to be interpolated have been detected as sectors and the degrees of freedom of the transformation have been properly distributed, a transformation matrix for that dimension can be calculated. A simple solution is to use the least squares (LS) approach from (16). However, since, in most cases, the true array response can only be estimated via empirical measurements that are corrupted by measurement errors the simple least squares approach is degraded. Alternatively, a total least squares (TLS) approach can be applied, as proposed in [26]. The transformation matrix can be calculated by finding the TLS solution to

$$([\mathcal{A}]_{(r)} + \mathbf{E})^{\mathrm{H}} \tilde{\mathbf{B}}_r = (\bar{\mathbf{A}}_r + \mathbf{F})^{\mathrm{H}}, \tag{31}$$

where $\mathbf{E}$ and $\mathbf{F}$ are matrices containing the measurement errors. A final transformation matrix $\mathbf{B}_r$ can be obtained by

$$\mathbf{B}_r = \tilde{\mathbf{B}}_r^{\mathrm{H}}. \tag{32}$$

This step will only improve the results over the standard LS formulation (12) if the knowledge of $\mathcal{A}$ is imperfect. On the other hand the TLS formulation is equivalent to the LS formulation when $\mathcal{A}$ is perfectly known. Therefore, applying the TLS will never result in performance degradation when compared to the LS. The TLS approach does however come at the cost of increased computational complexity when compared to the LS formulation.

C. Data Transformation and DOA Estimation

After $\mathbf{B}_r$ has been obtained for all the dimensions to be interpolated the received data can be transformed by

$$\mathcal{X} \times_1 \mathbf{B}_1 \times_2 \mathbf{B}_2 \ldots \times_R \mathbf{B}_R. \tag{33}$$

ISBN 978-3-8007-3662-1

$$\mathcal{S}_k^{(r)} = \begin{cases} \left\{ \left\lceil c_k^{(r)} - \frac{\Xi \xi_k^{(r)}}{2} \right\rfloor_{\mathcal{D}_\Delta}, \left\lceil c_k^{(r)} - \frac{\Xi \xi_k^{(r)}}{2} + \Delta \right\rfloor_{\mathcal{D}_\Delta}, ..., \left\lceil c_k^{(r)} + \frac{\Xi \xi_k^{(r)}}{2} \right\rfloor_{\mathcal{D}_\Delta} \right\} & , \Xi \xi_k^{(r)} < \left| u_k^{(r)} - l_k^{(r)} \right| \\ \left\{ l_k^{(r)}, l_k^{(r)} + \Delta, ..., u_k^{(r)}, \ \Xi \xi_k \right\} \geq \left| u_k^{(r)} - l_k^{(r)} \right| \end{cases} \quad (24)$$

The transformation also affects the noise component, leading to colored noise at the output. This requires some sort of prewhitening to be applied prior to some DOA estimation algorithms. For prewhitening schemes, we refer to [23] for the matrix case and [24] for the tensor case.

Depending on the structure chosen for the interpolated array many different DOA estimation methods can be applied. Multidimensional ESPRIT approaches such as [22] or methods based on the PARAFAC decomposition such as [27], [8] can be used.

V. Numerical Results

In order to show the performance of the proposed method a set of numerical simulations is performed. Three signals impinge on a 8×8 planar array. The spatial frequencies are set to $(-\frac{\pi}{3}, \frac{\pi}{3})$, $(\frac{\pi}{5}, \frac{\pi}{5})$ and $(\frac{\pi}{3}, -\frac{\pi}{3})$. All three signals have a correlation coefficient of 0.8 with each other. As shown in (8) and in (11), the array response is corrupted by inducing a random phase delay at each sensor element. In this section, we assume a normal distribution for these phase delays and our goal is to compensate these phase delays by applying the proposed multidimensional interpolation scheme. Additionally, an extra unknown phase delay with standard deviation equal to $\frac{1}{10}$ of the standard deviation of the known errors is introduced. The proposed technique is compared to the results of an array with no errors introduced (perfect array), to the interpolation method from [19] using a matrix approach extending the optimization problem to two dimensions, and to the corrupted array when no interpolation method is applied. For the matrix method, the transformation matrix is calculated according to (31) taking into account $[\mathcal{A}]_{(R+1)}$ and $\bar{\mathbf{A}}$, that is, the total steering matrices are used. The DOAs are estimated using the PARAFAC decomposition to estimate the factors and the translation invariance for DOA estimation. Estimations are performed using $N = 100$ and the final RMSE is the average with respect to 1000 Monte Carlo runs.

Figure 3 presents the results for phase errors following a normal distribution with standard deviation of $\frac{\pi}{20}$ radians. The results show that, for low SNRs where the noise is the main source of bias, the proposed technique is able to provide results similar to those obtained with a perfect array. However, the non interpolated array is subject to a constant bias due to the phase delay present at the sensor elements.

Figure 4 presents the results for phase errors following a normal distribution with standard deviation of $\frac{\pi}{10}$ radians. The performance of the proposed interpolation method with comparison to the perfect array worsens as the errors get larger. Also, it is noticeable that the proposed method introduces a bias that saturates the accuracy of the estimation

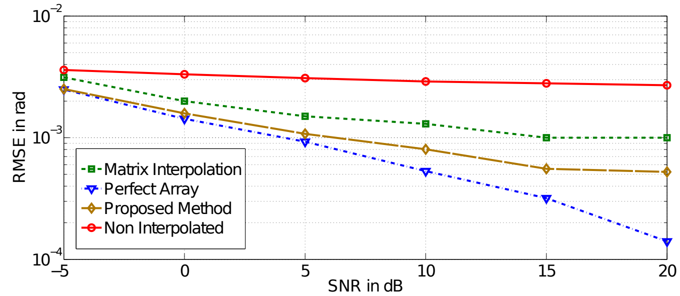

Figure 3: Results for a standard deviation of $\frac{\pi}{20}$ radians

approximately at the 5 dB mark. This bias is caused by the transformation error.

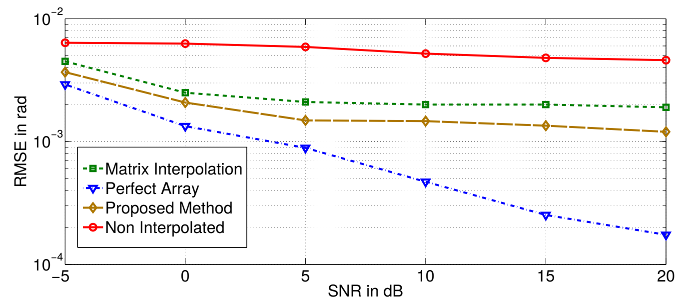

Figure 4: Results for a standard deviation of $\frac{\pi}{10}$ radians

Finally, Figure 5 presents the results for phase errors following a normal distribution with standard deviation of $\frac{\pi}{5}$ radians. The performance of the proposed method worsens considerably, as it becomes hard to compensate for the large phase errors introduced by each sensor elements. However, the performance of the proposed method is still superior to the estimation done without employing array interpolation.

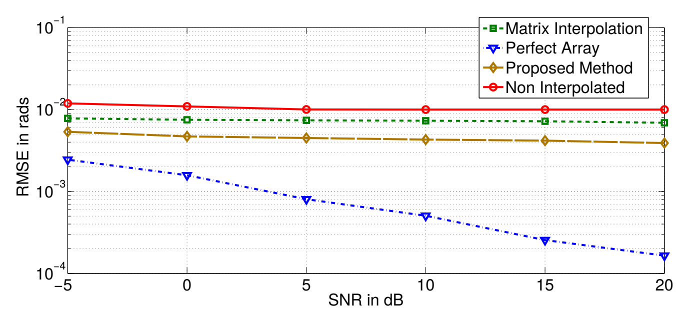

Figure 5: Results for a standard deviation of $\frac{\pi}{5}$ radians

In all simulations the proposed method outperformed its matrix counterpart. In the proposed method, the interpolation matrices are calculated for each dimension in a independent way. In the matrix approach, however, the dimensions are not decoupled and a total transformation matrix must be calculated taking into account all the permutations possible between angular responses from each dimension. Thus, the calculation of this transformation matrix is highly overdetermined resulting in a larger transformation error. Another advantage of the proposed method is that the optimization problems are one dimensional, resulting in a smaller computational load.

VI. Conclusion

In this work a method for interpolating multidimensional arrays was proposed. The approach decouples the problem of array interpolation and calculates a robust transformation matrix by means of solving an optimization problem based on array transformation errors and also by utilizing the TLS approach. The performance of the proposed method is studied by means of numerical simulations. It is shown that the method is capable of compensating for moderate phase errors introduced by the sensor elements of the array.

Acknowledgments

The research leading to the results reported in this paper has received funding from Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) under the PVE grant number 88881.030392/2013-01. This support is greatly acknowledged.

References

[1] Y. Bresler and A. Macovski, “Exact Maximum Likelihood Estimation of Superimposed Exponentials Signals in Noise,” *IEEE ASSP Magazine*, vol. 34, pp. 1081–189, 1986.

[2] P. Stoica and A. Nehorai, “A Novel Eigenanalysis Method for Direction Estimation,” in *Proceedings IEEE F.*, 1990.

[3] A. J. Barabell, “Improving the Resolution Performance of Eigenstructured Based Direction-Finding Algorithms,” in *Proceedings of ICASSP 83*, 1983.

[4] J. E. Evans, J. R. Johnson, and D. F. Sun, “Application of advanced signal processing techniques to angle of arrival estimation in ATC navigation and surveillance system,” Massachusetts Institute of Technology, Tech. Rep., 1982.

[5] S. Pillai and B. H. Kwon, “Forward/backward Spatial Smoothing Techniques for Coherent Signal Identification ,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, pp. 8–9, January 1989.

[6] R. Roy and T. Kailath, “ESPRIT - estimation of signal parameters via rotation invariance techniques,” *IEEE Transactions on Acoustics Speech and Signal Processing*, vol. 17, 1989.

[7] N. D. Sidiropoulos, R. Bro, and G. B. Giannakis, “Parallel factor analysis in sensor array processing,” *Signal Processing, IEEE Transactions on*, vol. 48, no. 8, pp. 2377–2388, 2000.

[8] J. P. C. L. da Costa, F. Roemer, M. Weis, and M. Haard, “Robust R-D parameter estimation via closed-form PARAFAC,” in *Proc. ITG Workshop on Smart Antennas (WSA’10)*, 2010.

[9] T. Bronez, “Sector interpolation of non-uniform arrays for efficient high resolution bearing estimation,” in *Acoustics, Speech, and Signal Processing, 1988. ICASSP-88., 1988 International Conference on*, 1988.

[10] B. Friedlander and A. Weiss, “Direction finding using spatial smoothing with interpolated arrays,” *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 28, pp. 574–587, 1992.

[11] B. Friedlander, “The root-MUSIC algorithm for direction finding with interpolated arrays,” *Signal Processing*, vol. 30, pp. 15–29, 1993.

[12] M. Pesavento, A. Gershman, and Z.-Q. Luo, “Robust array interpolation using second-order cone programming,” *Signal Processing Letters*, vol. 9, pp. 8–11, 2002.

[13] M. Buhren, M. Pesavento, and J. F. Böhme, “Virtual array design for array interpolation using differential geometry,” in *Acoustics, Speech, and Signal Processing, 2004. Proceedings. (ICASSP ’04). IEEE International Conference on*, vol. 2, 2004.

[14] B. Lau, G. Cook, and Y. Leung, “An improved array interpolation approach to DOA estimation in correlated signal environments,” in *Acoustics, Speech, and Signal Processing, 2004. Proceedings. (ICASSP ’04). IEEE International Conference on*, 2004.

[15] B. Lau, M. Viberg, and Y. Leung, “Data-adaptive array interpolation for DOA estimation in correlated signal environments,” in *Acoustics, Speech, and Signal Processing, 2005. Proceedings. (ICASSP ’05). IEEE International Conference on*, 2005.

[16] R. O. Schmidt, “Multiple emitter location and signal parameter estimation,” *IEEE Transactions on Antennas and Propagation*, vol. 34, pp. 276–280, 1986.

[17] M. Buhren, M. Pesavento, and J. F. Bohme, “A new approach to array interpolation by generation of artificial shift invariances: interpolated ESPRIT,” in *Acoustics, Speech, and Signal Processing, 2003. Proceedings. (ICASSP ’03). 2003 IEEE International Conference on*, 2003.

[18] A. Weiss and M. Gavish, “The interpolated ESPRIT algorithm for direction finding,” in *Electrical and Electronics Engineers in Israel, 1991. Proceedings., 17th Convention of*, 1991.

[19] M. A. M. Marinho, F. Antreich, J. P. C. L. da Costa, and J. A. Nossek, “A Signal Adaptive Array Interpolation Approach with Reduced Transformation Bias for DOA estimation of Highly Correlated Signals,” in *Acoustics, Speech, and Signal Processing, 2004. Proceedings. (ICASSP ’14). IEEE International Conference on*, 2014.

[20] F. Roemer, “Advanced algebraic concepts for efficient multi-channel signal processing,” Ph.D. dissertation, TU Ilmenau, 2012.

[21] J. P. C. L. da Costa, F. Roemer, M. Haardt, and R. T. de Sousa Jr., “Multi-Dimensional Model Order Selection,” *EURASIP Journal on Advances in Signal Processing*, vol. 26, 2011.

[22] M. Haardt, F. Roemer, and G. D. Gado, “Higher-order SVD based subspace estimation to improve parameter estimation accuracy in multidimensional harmonic retrieval problems,” *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3198 – 3213, July 2008.

[23] J. P. C. L. da Costa, F. Roemer, and M. Haardt, “Deterministic prewhitening to improve subspace parameter estimation techniques in severely colored noise environments,” in *Proc. 54th International Scientific Colloquium (IWK)*, 2009.

[24] J. P. C. L. da Costa, K. Liu, H. C. So, F. Roemer, M. Haardt, and S. Schwarz, “Generalized multidimensional prewhitening for enhanced signal reconstruction and parameter estimation,” in *Signal Processing, Elsevier publisher*, 2013.

[25] M. A. M. Marinho, J. P. C. L. da Costa, F. Antreich, and L. R. A. X. de Menezes, “Unscented transformation based array interpolation,” in *Proc. 40th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Brisbane, Australia, April 2015.

[26] M. A. M. Marinho, F. Antreich, J. P. C. L. da Costa, and J. A. Nossek, “Reduced Rank TLS Array Interpolation for DOA Estimation,” in *18th International ITG Workshop on Smart Antennas (WSA 2014)*, 2014.

[27] J. P. C. L. da Costa, D. Schulz, F. Roemer, M. Haardt, and J. A. A. Jr., “Robust R-D Parameter Estimation via Closed-Form PARAFAC in Kronecker Colored Environments,” in *Proc. 7-th International Symposium on Wireless Communications Systems (ISWCS 2010)*, 2010.

ISBN 978-3-8007-3662-1