Antenna Array Based Positioning Scheme for Unmanned Aerial Vehicles

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***Abstract*—Recently antenna arrays have been incorporated in Unmanned Aerial Vehicles (UAVs) in order to improve their communications capacities. Such antenna arrays can be further exploited in order to provide accurate estimation of a UAV pose and attitude which are necessary for the UAV movement and control. In this paper, we propose a pose estimation solution based on the 2-D Standard ESPRIT with Forward Backward Averaging (FBA) for determining the directions of arrival (DOAs) of the incoming signals. Moreover, by exploiting the geometry of the antenna array, it is possible to estimate the antenna positions, and then, by applying the TRIAD algorithm, it is possible to compute the attitude. We show, by means of simulations, that our proposed solution provides a very accurate pose estimation.**

***Index Terms*—Unmanned Aerial Vehicles, Direction of Arrival, Attitude Estimation, ESPRIT**

I. Introduction

Recently UAVs have been employed to assist in various applications, from military to civilian usage, due to the possibility of performing hazardous tasks without endangering human life and generally possessing lower costs of operation. However, the pose information is crucial for the displacement and control of the UAVs. Attitude estimation can also be performed by using Inertial Measurement Units (IMUs) such as accelerometers and gyroscopes. These units, however, suffer from imprecision caused by measurement and drift errors [1], [2]. Position estimation is usually done with the use of Global Positioning System (GPS), the satellite information can also be used to provide estimation for the attitude as presented on [3], [4], however, such solutions require additional hardware.

Taking advantage of the fact that antenna arrays are currently being implemented into UAVs to enhance communication systems we can use the available structure to perform the pose estimation. Thus, no additional hardware is necessary at the UAV, not increasing the vehicle weight and, allowing the position estimating to be done in situations where the GPS is not available.

In [2], an antenna array based attitude estimator has been first proposed. As a drawback of [2], the prior knowledge of the yaw is required and moreover the proposed solution is severely degraded in scenarios with multipath components. Although, in [5], the constraint of yaw knowledge is relaxed and the solution is extended for multipath scenarios, the numerical precision of the solution has a lower bound even for noiseless scenarios. On [6], the usage of an ESPAR antenna array is suggested to achieve improved estimation accuracy.

In this paper, we propose a solution assuming that both transmit and receive sides are equipped with antenna arrays, i.e. a MIMO system. In such scenarios, it is possible not only to increase the accuracy of the attitude estimation, but also to estimate the position of the UAV in space without the use of a GPS system.

The remainder of this paper is divided into four more sections. In Section II the system model of the antenna array based positioning scheme is explained. In Section III, the proposed solution is described. In Section IV simulation results are presented. Conclusions are drawn in Section V.

II. System Model

A. Scenario Description

We assume a MIMO system composed of a uniform rectangular array (URA) at the base station and an antenna array at the UAV as shown in Figure 1. The URA is the center of the 3-D space formed by x, y and z. The UAV is endowed with d transmitting antennas and the URA at the base station in Figure 1 has size M by M, where $M \geqslant d$.

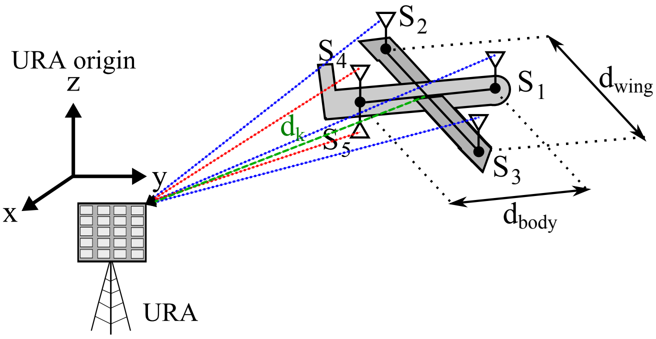

Figure 1: System model composed of a URA at the base station and an antenna array at the UAV.

For the sake of simplicity, the URA is considered to be small enough to be represented as a point in space. We place the origin of a 3-D coordinate system at the point defined by the position of the URA. The antennas at the UAV are

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also considered to be points in space, and are denoted as $P_{s_1} \dots P_{s_d} \in \mathbb{R}^3$. The distance any two antennas i and j, $\left\|\overrightarrow{P_{s_i}P_{s_j}}\right\|$, is considered to be known.

B. Attitude Angles Definition

Figure 2 illustrates a graphical representation of the definitions of pitch, yaw and roll for this work. Let us define the pitch as the line that passes trough the nose and tail antennas, a wing axis as the line that passes through both wing antennas and the yaw axis as a line perpendicular to these two and passing through the center of the UAV. The attitude angles are defined as the rotation of the plane along these axes in relation to a given inertial frame of reference.

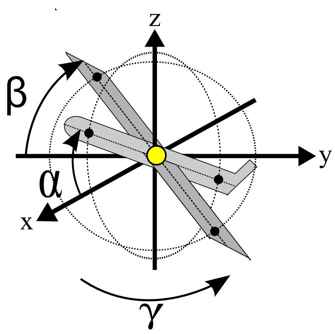

Figure 2: Definitions of pitch α, yaw β and roll γ

C. Data Model

The signal received at the m_1, m_2 antenna at the base station at a time snapshot t can be represented as:

$$y_{m_1,m_2}(t) = \sum_{i=1}^{k} s_i(t) \prod_{r=1}^{2} e^{j\cdot(m_r-1)\cdot\mu_i^{(r)}} + n_{m1,m2}(t), \quad (1)$$

where $s_i(t)$ is the complex symbol transmitted by the i-th source at time snapshot t, $n_{m_1,m_2,t}$ is the zero mean additive white Gaussian noise present at the antenna m_1, m_2 at time snapshot t. $\mu_i^{(r)}$ represents the spatial frequency of the signal transmitted by the i-th source over the r-th dimension. For example, the spatial frequencies of a signal impinging over the URA presented in Figure 3 are given by

$$\mu_i^{(1)} = 2\pi\frac{\Delta}{\lambda}\cos(\theta_i)\sin(\phi_i), \quad (2)$$

$$\mu_i^{(2)} = 2\pi\frac{\Delta}{\lambda}\sin(\theta_i)\sin(\phi_i), \quad (3)$$

where θ_i and ϕ_i are the azimuth and elevation of arrival of the i-th signal, Δ is the separation between antenna elements and λ is the wavelength of the incoming signal.

Equation (1) can be rewritten in matrix notation as

$$\boldsymbol{X} = \boldsymbol{A}\cdot\boldsymbol{S} + \boldsymbol{N}, \quad (4)$$

where $\boldsymbol{S}$ is the symbol matrix, whose elements are $s_i(t)$, T is the number of time snapshots, $\boldsymbol{A}$ is the steering matrix whose elements are $e^{j\cdot(m_r-1)\cdot\mu_i^{(r)}}$. The matrix $\boldsymbol{N} \in \mathbb{C}^{M^2\times T}$ contains the noise samples. $\boldsymbol{X} \in \mathbb{C}^{M^2\times T}$ is the measurement matrix.

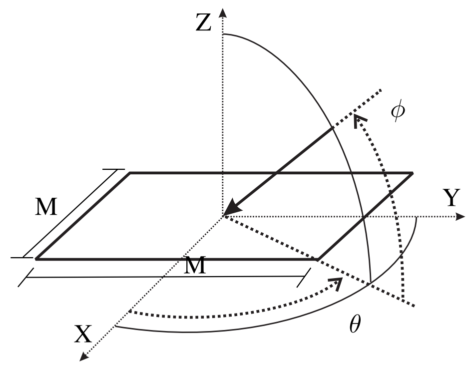

Figure 3: Graphical representation of a $M \times M$ URA

III. Proposed Antenna Array based Positioning Scheme

A. Estimation of θ and ϕ

According to Figure 3 the DOA is composed of two parameters: the elevation ϕ and the azimuth θ. Our goal in this first part is to estimate ϕ_i and θ_i, which are the DOAs related to the i-th antenna at the UAV. ESPRIT is a DOA estimation scheme originally proposed in [7] for one dimensional arrays. Extensions exist for multidimensional data for the matrix case [8] and for the tensor case [9]. Since the R-D Standard ESPRIT (SE) is a closed-form scheme [10], it is selected to estimate the DOAs incident at the URA at the base station. Also as proposed in [10], we incorporate the Forward Backward Averaging (FBA) in order to further increase the accuracy of the parameters.

The first step into estimating the DOAs is calculating the sample covariance matrix given by

$$\hat{\boldsymbol{R}}_{\boldsymbol{XX}} = \frac{1}{T}\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}} \in \mathbb{C}^{M^2\times M^2}, \quad (5)$$

where $^{\mathrm{H}}$ represents the Hermitian operator.

Next, we apply the eigenvalue decomposition to the sample covariance matrix

$$\hat{\boldsymbol{R}}_{\boldsymbol{XX}} = \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{E}^{\mathrm{H}}. \quad (6)$$

The signal subspace $\boldsymbol{E}_S$ of $\boldsymbol{X}$ can be constructed by selecting only the singular vectors related to the d largest singular values, the remaining singular vectors form the noise subspace $\boldsymbol{E}_N$ of $\boldsymbol{X}$. In our case the model order d is known *a priori* since the number of antennas at the UAV is known, if the model order is unknown it can be estimated using model order selection schemes such as the ones presented in [11], [12].

ESPRIT relies on the Shift Invariance principle, which means that an array can be divided into two subarrays that differ only by an offset. As presented in the data model (1) the signal received over the different antennas differs only by a phase delay.

Figure 4 presents the 1-D selection matrices $\boldsymbol{J}_i^{(r)} \in \mathbb{R}^{M^{(\mathrm{sel})}\times M}$, $i = 1, 2$, these matrices are used to select different sets of $M^{(\mathrm{sel})}$ out of M elements in order to apply the shift invariance equations.

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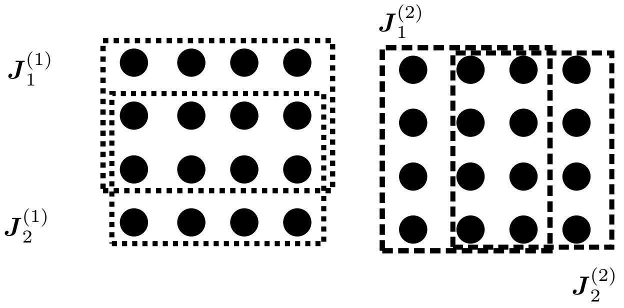

Figure 4: Graphical representation of selection matrices

$$\begin{aligned}\boldsymbol{J}_1^{(r)} &= [\boldsymbol{I}_{M^{(\text{sel})}} \boldsymbol{0}_{M^{(\text{sel})}\times 1}],\\ \boldsymbol{J}_2^{(r)} &= [\boldsymbol{0}_{M^{(\text{sel})}\times 1} \boldsymbol{I}_{M^{(\text{sel})}}].\end{aligned} \tag{7}$$

Using the 1-D selection matrices the R-D selection matrices can be constructed as

$$\begin{aligned}\tilde{\boldsymbol{J}}_1^{(r)} &= \boldsymbol{I}_M \otimes \boldsymbol{J}_1^{(r)} \otimes \boldsymbol{I}_M,\\ \tilde{\boldsymbol{J}}_2^{(r)} &= \boldsymbol{I}_M \otimes \boldsymbol{J}_2^{(r)} \otimes \boldsymbol{I}_M.\end{aligned} \tag{8}$$

A set of shift invariance equations can be constructed for the matrix representation as

$$\begin{aligned}\tilde{\boldsymbol{J}}_1^{(1)} \cdot \boldsymbol{A} \cdot \boldsymbol{\Phi}^{(1)} &= \tilde{\boldsymbol{J}}_2^{(1)} \cdot \boldsymbol{A},\\ \tilde{\boldsymbol{J}}_1^{(2)} \cdot \boldsymbol{A} \cdot \boldsymbol{\Phi}^{(2)} &= \tilde{\boldsymbol{J}}_2^{(2)} \cdot \boldsymbol{A},\end{aligned} \tag{9}$$

where the matrices $\boldsymbol{\Phi}^{(r)}$ are given by

$$\boldsymbol{\Phi}^{(r)} = \text{diag}\{[e^{j\mu_1^{(r)}}, e^{j\mu_2^{(r)}}, \ldots, e^{j\mu_d^{(r)}}]\}. \tag{10}$$

The matrix $\boldsymbol{A}$ is not known, however, in the absence of noise, the d columns of $\boldsymbol{A}$ and the d columns of the signal subspace $\boldsymbol{E}_S$ span the same subspace, and are related by a non singular transform matrix $\boldsymbol{T} \in \mathbb{C}^{d\times d}$ in the following way

$$\boldsymbol{A} = \boldsymbol{E}_S \cdot \boldsymbol{T}. \tag{11}$$

Thus, (9) can be rewritten as

$$\begin{aligned}\tilde{\boldsymbol{J}}_1^{(1)} \cdot \boldsymbol{E}_S \cdot \boldsymbol{\Psi}^{(1)} &\approx \tilde{\boldsymbol{J}}_2^{(1)} \cdot \boldsymbol{E}_S,\\ \tilde{\boldsymbol{J}}_1^{(2)} \cdot \boldsymbol{E}_S \cdot \boldsymbol{\Psi}^{(2)} &\approx \tilde{\boldsymbol{J}}_2^{(2)} \cdot \boldsymbol{E}_S,\end{aligned} \tag{12}$$

where $\boldsymbol{\Psi}^{(r)}$ is related to $\boldsymbol{\Phi}^{(r)}$ by

$$\boldsymbol{\Psi}^{(r)} = \boldsymbol{T} \cdot \boldsymbol{\Phi}^{(r)} \cdot \boldsymbol{T}^{-1}. \tag{13}$$

This transformation does not change the eigenvalues, hence, the eigenvalues of $\boldsymbol{\Psi}^{(r)}$ are equal to $e^{j\mu_1^{(r)}}, e^{j\mu_2^{(r)}}, \ldots, e^{j\mu_d^{(r)}}$. Solving the equation system presented in (12) the spatial frequencies of the d signals can be estimated, and thus yields an estimation of the DOAs.

B. Direction Vector Computation

Once the DOAs have been estimated, the next step is to generate a direction vector for the line representing the signals. The center of the URA is considered to be the origin of our coordinate system and shall be denoted as O. The URA is made small enough so that to the incoming wave front can be represented as a set of straight lines. Antennas are considered as points in the space and are contained within the line representing the signal they irradiate. Since the URA is the origin of the coordinate system, all signal lines pass through the origin, thus an equation representing the coordinates x_{P_i}, y_{P_i} and z_{P_i} of any point P_i within the line representing the i-th signal can be written as

$$\begin{aligned}x_{P_i} &= \|m_{P_i}\| \cdot \sin(\phi_i)\cos(\theta_i),\\ y_{P_i} &= \|m_{P_i}\| \cdot \sin(\phi_i)\sin(\theta_i),\\ z_{P_i} &= \|m_{P_i}\| \cdot \cos(\phi_i),\end{aligned} \tag{14}$$

where θ_i and ϕ_i are the azimuth and elevation of the i-th signal respectively and $\|m_{P_i}\| \in \mathbb{R}$ is the magnitude of the vector $\overrightarrow{OP_i}$.

C. Position Estimation

After obtaining the direction vector for all signals,position estimates of the antennas on the coordinate system can be obtained by solving a system of equations based on the distance between each pair of antennas. The Euclidean distance between two points $A \in \mathbb{R}^3$ and $B\in \mathbb{R}^3$ is given by

$$m_{\overrightarrow{AB}} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2}. \tag{15}$$

To find the estimate of the point where antennas i and j are located, x_A, x_B, y_A, y_B, z_A and z_B can be replaced by x_{P_i}, x_{P_j}, y_{P_i}, y_{P_j}, z_{P_i} and z_{P_j}, respectively. Since the distance between each pair of antenna and the DOA of each signal is known, $\|m_{P_i}\|$ and $\|m_{P_j}\|$ can be obtained, yielding the estimated positions of antennas i and j relative to the center of the URA. Note that we have $n(n-1)/2$ available equations to choose from, where n is the number of antennas at the UAV.

D. Calculation of Pitch, Yaw and Roll

Once antenna positions at the UAV have been estimated, it is possible to obtain the estimation of the attitude. The TRIAD [13], [14] algorithm allowed the development of early satellite navigation systems and still represents the state of practice for space and aircraft instrument based attitude estimation. It involves only two linear independent reference vectors and their respective measured directions.

By defining two linearly independent reference vectors

$$\begin{aligned}\vec{R}_1 &= [x_{\vec{R}_1}, y_{\vec{R}_1}, z_{\vec{R}_1}]^T,\\ \vec{R}_2 &= [x_{\vec{R}_2}, y_{\vec{R}_2}, z_{\vec{R}_2}]^T,\end{aligned} \tag{16}$$

and their respective measured direction vectors

$$\begin{aligned}\vec{r}_1 &= [x_{\vec{r}_1}, y_{\vec{r}_1}, z_{\vec{r}_1}]^T,\\ \vec{r}_2 &= [x_{\vec{r}_2}, y_{\vec{r}_2}, z_{\vec{r}_2}]^T.\end{aligned} \tag{17}$$

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The TRIAD algorithm tries to find the rotation matrix $\boldsymbol{B}$ that satisfies

$$\vec{r}_i = \boldsymbol{B}\vec{R}i, \tag{18}$$

where $\boldsymbol{B}^T\boldsymbol{B} = \boldsymbol{I}$ and $\det(\boldsymbol{B}) = \pm 1$, i.e $\boldsymbol{B}$ is a orthogonal matrix and preserves the magnitude of any vector $\vec{R}i$ it operates on. Since $\vec{R}_1$ and $\vec{R}_2$ are linearly independent vectors, a linear independent vector orthogonal to both can by obtained by

$$\vec{R}_3 = \vec{R}_1 \times \vec{R}_2, \tag{19}$$

where $\times$ is the vector cross product operation, the same can be done for $\vec{r}_1$ and $\vec{r}_2$

$$\vec{r}_3 = \vec{r}_1 \times \vec{r}_2. \tag{20}$$

Since a rotation applied to $\vec{R}_1$ and $\vec{R}_2$ would also rotate $\vec{R}_3$, a linear system can be written as

$$[\vec{r}_1, \vec{r}_2, \vec{r}_3] = \boldsymbol{B}[\vec{R}_1, \vec{R}_2, \vec{R}_3], \tag{21}$$

here the commas represent the concatenation of two column vectors. In the noise free case the present linear system holds exact and will yield an orthogonal matrix $\boldsymbol{B}$. However, in the presence of noise, the result might be a non orthogonal matrix. To address this problem the TRIAD algorithm replaces $\vec{R}_1$, $\vec{R}_2$ and $\vec{R}_3$ by

$$\begin{aligned}\vec{S}_1 &= \frac{\vec{R}_1}{\left\|\vec{R}_1\right\|},\\ \vec{S}_2 &= \frac{\vec{R}_1 \times \vec{R}_2}{\left\|\vec{R}_1 \times \vec{R}_2\right\|},\\ \vec{S}_3 &= \frac{\vec{S}_1 \times \vec{S}_2}{\left\|\vec{S}_1 \times \vec{S}_2\right\|},\end{aligned} \tag{22}$$

respectively. We also replace $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ by

$$\begin{aligned}\vec{s}_1 &= \frac{\vec{r}_1}{\|\vec{r}_1\|},\\ \vec{s}_2 &= \frac{\vec{r}_1 \times \vec{r}_2}{\|\vec{r}_1 \times \vec{r}_2\|},\\ \vec{s}_2 &= \frac{\vec{s}_1 \times \vec{s}_2}{\|\vec{s}_1 \times \vec{s}_2\|}.\end{aligned} \tag{23}$$

respectively. Note that by construction the matrices $[\vec{S}_1, \vec{S}_2, \vec{S}_3]$ and $[\vec{s}_1, \vec{s}_2, \vec{s}_3]$ are orthogonal matrices, since their columns are made orthogonal to each other. This avoids the computationally intensive task of calculating the matrix inverse, since $[\vec{S}_1, \vec{S}_2, \vec{S}_3]^{-1} = [\vec{S}_1, \vec{S}_2, \vec{S}_3]^T$. Thus, an estimate of $\boldsymbol{B}$ can be found by

$$\hat{\boldsymbol{B}} = [\vec{s}_1, \vec{s}_2, \vec{s}_3] \cdot [\vec{S}_1, \vec{S}_2, \vec{S}_3]^T. \tag{24}$$

As the rotation matrix is directly dependent on the pitch, yaw and roll an estimation can be extracted from $\hat{\boldsymbol{B}}$. The yaw, pitch and roll rotation matrices are given by

$$\begin{aligned}\boldsymbol{R}(\alpha) &= \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix},\\ \boldsymbol{R}(\beta) &= \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix},\\ \boldsymbol{R}(\gamma) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{bmatrix}.\end{aligned} \tag{25}$$

If the rotation follows the order of yaw, pitch and roll, then

$$\hat{\boldsymbol{B}} = \boldsymbol{R}(\alpha)\boldsymbol{R}(\beta)\boldsymbol{R}(\gamma)$$

$$\hat{\boldsymbol{B}} = \begin{bmatrix} \cos(\hat{\alpha})\cos(\hat{\beta}) & \cos(\hat{\alpha})\sin(\hat{\beta})\sin(\hat{\gamma}) - \sin(\hat{\alpha})\cos(\hat{\gamma}) & \cos(\hat{\alpha})\sin(\hat{\beta})\cos(\hat{\gamma}) + \sin(\hat{\alpha})\sin(\hat{\gamma}) \\ \sin(\hat{\alpha})\cos(\hat{\beta}) & \sin(\hat{\alpha})\sin(\hat{\beta})\sin(\hat{\gamma}) - \cos(\hat{\alpha})\cos(\hat{\gamma}) & \sin(\hat{\alpha})\sin(\hat{\beta})\cos(\hat{\gamma}) + \cos(\hat{\alpha})\sin(\hat{\gamma}) \\ -\sin(\hat{\beta}) & \cos(\hat{\beta})\sin(\hat{\gamma}) & \cos(\hat{\beta})\cos(\hat{\gamma}) \end{bmatrix} \tag{26}$$

where $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are estimates of the yaw pitch and roll respectively. Note that changing the order of rotation will change the structure of $\hat{\boldsymbol{B}}$.

The TRIAD algorithm allows an estimate of the UAV attitude to be obtained by obtaining only two linearly independent position vectors, such as a wing and the nose position vectors. Other methods for estimating the attitude exist, such as the QUEST [15] and SVD based methods such as [16]. These methods are capable of taking into account a broad set of measurements and reference vectors resulting in a more accurate estimation. However, they are more computationally bulky than the TRIAD algorithm and, usually, slower, thus not practical for real-time estimation scenarios.

IV. Experiments

The first result analyzed is the accuracy of the DOA estimation provided by the ESPRIT algorithm. Figure 5 presents the RMSE of the DOA estimation versus the SNR for a URA of size 10×10 with inter element separation of $\frac{\lambda}{2}$. Two signals present and each estimation is made with 50 snapshots, i.e $T = 50$. Even for negative SNR scenarios the estimation yields results with a precision in the order of 10^{-2} degrees.

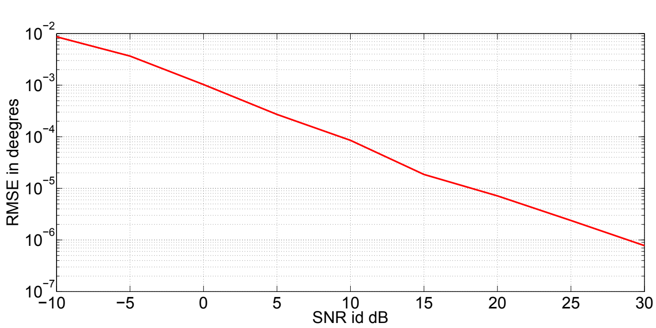

Figure 5: RMSE of DOA estimation in degrees

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The high accuracy provided by the ESPRIT algorithm is capable of yielding very precise results for the position estimation when the UAV is located relatively close to the receiving URA. However, as the distance increases, the variations in the DOAs of the received signal become very small, thus, even very small errors in the estimation will yield large errors in the estimate of the position of the UAV. Figure 6 shows how the error varies according to the distance between the UAV and the URA. The SNR for this simulation is fixed at 10 dB.

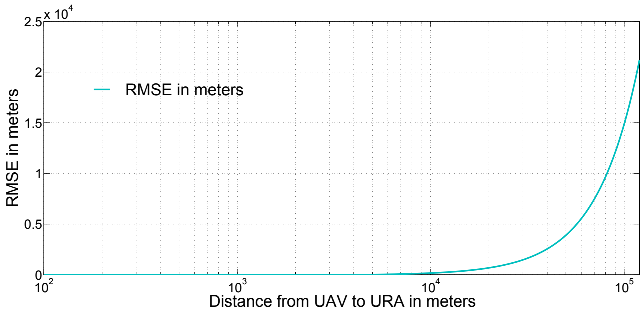

Figure 6: RMSE of estimated position in relation to the distance between the UAV and the URA

Figure 7 presents a comparison between the proposed method and [5]. In the simulation the UAV is placed at 1000 m from the base station and the SNR is 10 dB. Note that the high accuracy of the ESPRIT already presented in Figure 5 yields a very accurate estimation of the UAV attitude.

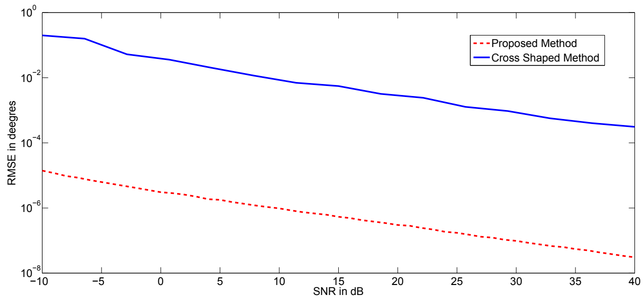

Figure 7: RMSE of estimates of the pitch, yaw and roll versus SNR

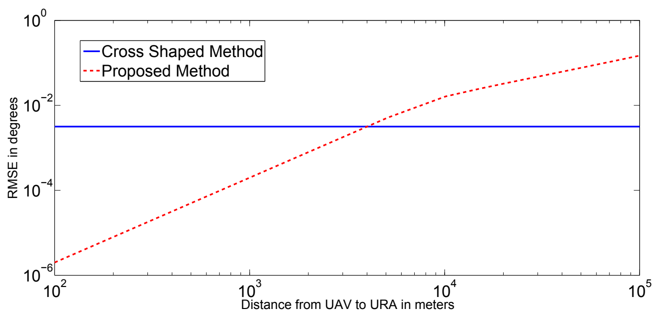

Figure 8: RMSE of estimated attitude versus distance from the URA

Finally, Figure 8 presents a comparison between the RMSE of the estimated attitude for both techniques versus the distance from the URA at the base station. The SNR is kept fixed for all distances in order to compare only the impact of the distance in the attitude estimation. The accuracy of the cross shaped method proposed on [5] is not affected by the distance between the base station and the UAV as long as the SNR remains unchanged, on the other hand, as the distance between the base station and the UAV increases the accuracy of the method proposed in this paper degrades. An alternative would be to choose the proper estimation method depending on the distance between the UAV and base station, choosing the more accurate one depending on the distance.

V. Conclusion

In this paper we have proposed to apply an antenna array communication system to estimate the position and the attitude of a UAV. The proposed method is capable of accurately estimating the position and attitude of an UAV in space without requiring prior knowledge of pitch, yaw or roll. Also, a GPS equipment is not necessary and the distance from UAV to base station can be easily obtained once the position has been estimated.

Results show that the technique is robust and capable of performing in a robust manner even at low SNR scenarios. For long distances between the UAV and the URA the proposed technique becomes inaccurate due to the fact that very small variations in the estimated DOAs result in large positioning errors.

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