Improved Array Interpolation for Reduced Bias in DOA Estimation for GNSS

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BIOGRAPHY

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1 INTRODUCTION

Vandermonde or centro-hermitian array structures are of special interest in DOA estimation since they allow for reduced computational complexity with fast converging methods or even closed-form solutions. Popular methods such as Iterative Quadratic Maximum Likelihood (IQML) [1], Root-WSF [2] and Root-MUSIC [3] all rely on a Vandermonde or centro-hermitian array response. Another important property of a centro-hermitian array response that it is allowing the application Spatial Smoothing (SPS) [4] and Forward Backward Averaging (FBA) [5]. These techniques enable the application of subspace based DOA estimation methods and precise model order estimation in the presence of highly correlated or even coherent signals. Dealing with highly correlated signals is of great importance when facing strong multipath scenarios or in case of safety-critical applications when specific jamming, meaconing, or spoofing is received by Global Navigation Satellite Systems (GNSS) receivers.
To obtain an array response that is Vandermonde or centro-hermitian is very hard in reality due to effects such as mutual coupling of the antennas, changes in antenna location, material tolerances, hardware biases, and the sur-

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rounding environment of the array. Even when the construction is possible there is no guarantee that the response of such an array will be kept invariant over time, e.g. due to wear and temperature stability. As a solution to these limitations array interpolation (mapping) was proposed [6] where an arbitrary array response is mapped onto the desired Vandermonde or centro-hermitian response. Most array interpolation schemes divide the complete angular region into limited angular sectors. For each sector a mapping/transformation matrix is defined using knowledge of the empirical measured array response. Then after transformation to a desired virtual array, FBA or SPS [7] and DOA estimation algorithms such as Root-MUSIC [8] can be applied. However, when performing array interpolation with a sector-by-sector processing the mapping matrices have to be carefully derived in order to minimize the transformation bias within each sector and on the other hand to control its out-of-sector response. The out-of-sector response was neglected in earlier works [7], [8], [9], [10]. Addressing the out-of-sector response by a signal adaptive weighting and a sector-by-sector estimation of highly correlated and closely spaced signal environments is proposed in [11] and [12]. Furthermore, although before the array interpolation the noise is white, after the array interpolation the noise becomes colored. Therefore, a prewhitening step is necessary for MUSIC [13] and Root-MUSIC algorithms [3],[8]. Such prewhitening would destroy the shift invariance properties necessary for the standard ESPRIT algorithm [14]. Array interpolation techniques that allow the application of a modified ESPRIT algorithm have been proposed in [15] and [16]. These techniques do not require the prewhitening step, thus allowing the direct application of the ESPRIT algorithm. However, they ignore the out-of-sector response and they do not consider the application of FBA or/and SPS and thus cannot be applied with highly correlated signals.

Another application of the array interpolation technique can be seen in [17] where the Vandermonde Invariance Transformation (VIT) was developed. The VIT does not try to address the physical imperfections of the array response but instead transforms the response of an array with a uniform Vandermonde response into one with a non uniform phase response. The VIT provides a noise shaping effect by lowering the noise power over a desired angular region and allowing a more precise DOA estimation at the cost of increased computational load.

In this work we do not apply the classical sector-by-sector mapping processing. Instead we propose a signal adaptive multi-sector array interpolation method that minimizes the transformation bias, allows dealing with highly correlated signals by applying FBA and/or SPS, and enables closed-form DOA estimation by ESPRIT with a generalized eigenvalue decomposition (GEVD). We derive a single mapping matrix that considers several angular sectors which include all the impinging signals of interest. These sectors are estimated and combined by applying a power scanning method of low complexity. The degrees of freedom of the mapping are distributed among the parts of the resulting combined sector following a simple adaptive weighting method, minimizing the transformation bias. The proposed approach avoids the out-of-sector response problems [11], [12] and allows to jointly estimate the DOAs of all impinging signals, while for the sector-by-sector processing for each sector a DOA estimation has to be performed, cf. [8], [9], [10] ,[11], [12], [15], [16]. Afterwards the VIT can be applied and followed by a second application of ESPRIT an additional gain of approx. 2 dB in DOA estimation performance can be achieved.

The proposed method can be applied to the vast majority of systems that rely on sensor arrays, e.g, radar systems, channel sounding and sonars. For this work we consider Global Navigation Satellite Systems (GNSS) and especially the case that highly correlated multipath or spoofing is received. Based on precise DOA estimation of all impinging signals specific Receiver Autonomous Integrity Monitoring (RAIM) algorithms allow reliable detection and mitigation of spoofing or coherent multipath signals in the receiver. Furthermore, precise DOA estimation also enables attitude estimation with high accuracy [18].

2 DATA MODEL

We consider a set of d wavefronts impinging onto an antenna array composed of M antenna elements. The received baseband signal after correlation can be expressed in matrix form as

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \in \mathbb{C}^{M \times N}, \quad (1)$$

where $\mathbf{S} \in \mathbb{C}^{d \times N}$ is the matrix containing the N symbols transmitted by each of the d sources, $\mathbf{N} \in \mathbb{C}^{M \times N}$ is the noise matrix with its entries drawn from $\mathcal{CN}(0, \sigma_n^2)$, and

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_d)] \in \mathbb{C}^{M \times d}, \quad (2)$$

where θ_i is the azimuth angle of the $i-$th signal and $\mathbf{a}(\theta_i) \in \mathbb{C}^{M \times 1}$ is the array response (empirical measurement).

The received signal covariance matrix $\mathbf{R_{XX}} \in \mathbb{C}^{M \times M}$ is given by

$$\mathbf{R_{XX}} = \mathrm{E}\{\mathbf{X}\mathbf{X}^{\mathrm{H}}\} = \mathbf{A}\mathbf{R_{SS}}\mathbf{A}^{\mathrm{H}} + \mathbf{R_{NN}}, \quad (3)$$

where $(\cdot)^{\mathrm{H}}$ stands for the conjugate transposition, and

$$\mathbf{R_{SS}} = \begin{bmatrix} \sigma_1^2 & \gamma_{1,2}\sigma_1\sigma_2 & \cdots & \gamma_{1,d}\sigma_1\sigma_d \\ \gamma_{1,2}^*\sigma_1\sigma_2 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & \\ \gamma_{1,d}^*\sigma_1\sigma_d & \gamma_{2,d}^*\sigma_2\sigma_d & \cdots & \sigma_d^2 \end{bmatrix}, \quad (4)$$

where σ_i^2 is the power of the $i-$th signal and $\gamma_{a,b} \in \mathbb{C}$, $|\gamma_{a,b}| \leq 1$ is the cross-correlation coefficient between signals a and b. $\mathbf{R_{NN}} \in \mathbb{C}^{M \times M}$ is a matrix with σ_n^2 over its diagonal and zeros elsewhere. An estimate of the signal covariance matrix can be obtained by

$$\hat{\mathbf{R}}_{\mathbf{XX}} = \frac{\mathbf{X}\mathbf{X}^{\mathrm{H}}}{N}. \tag{5}$$

3 ARRAY INTERPOLATION

Array interpolation is a set of techniques that aim to predict what signal would be received at an antenna array with a specific desired geometry based on the signal that was received by a real antenna array. In matrix form the transformation tries to achieve

$$\mathbf{B}\mathbf{A}_{\mathcal{S}} = \bar{\mathbf{A}}_{\mathcal{S}}, \tag{6}$$

where $\mathbf{A}_{\mathcal{S}}$ and $\bar{\mathbf{A}}_{\mathcal{S}}$ are array response matrices constructed considering the discrete set of angles

$$\mathcal{S} = \{l_{\mathcal{S}}, l_{\mathcal{S}} + \Delta, ..., u_{\mathcal{S}} - \Delta, u_{\mathcal{S}}\}. \tag{7}$$

Here, $l_{\mathcal{S}}$ is the lower bound, $u_{\mathcal{S}}$ is the upper bound of sector $\mathcal{S}$ and Δ is the angular resolution of the transformation. The matrix $\mathbf{B}$ can be seen as the matrix that achieves the best transform between a set of vectors $\mathbf{A}_{\mathcal{S}}$ and $\bar{\mathbf{A}}_{\mathcal{S}}$. If $\mathbf{A}_{\mathcal{S}}$ is error free $\mathbf{B}$ can be obtained by a least squares (LS) fit

$$\mathbf{B} = \bar{\mathbf{A}}_{\mathcal{S}}\mathbf{A}_{\mathcal{S}}^{\dagger} \in \mathbb{C}^{M \times M}, \tag{8}$$

where $(\cdot)^{\dagger}$ stands for the Moore–Penrose pseudo-inverse. Note that $\mathbf{B}$ is calculated differently from the usual linear regression formulation since it is obtained as to be multiplied by the right side of the original array response matrix, this is done so that $\mathbf{B}$ can then be applied directly on the estimated signal covariance matrix

$$\bar{\mathbf{R}}_{\mathbf{XX}} = \mathbf{B}\hat{\mathbf{R}}_{\mathbf{XX}}\mathbf{B}^{\mathrm{H}} \in \mathbb{C}^{M \times M}, \tag{9}$$

since

$$\begin{aligned} \mathbf{B}\mathbf{R}_{\mathbf{XX}}\mathbf{B}^{\mathrm{H}} &= \mathbf{B}\mathbf{A}\mathbf{S}\mathbf{S}^{\mathrm{H}}\mathbf{A}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}} + \mathbf{B}\mathbf{N}\mathbf{N}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}} \\ &= \bar{\mathbf{A}}\mathbf{S}\mathbf{S}^{\mathrm{H}}\bar{\mathbf{A}}^{\mathrm{H}} + \mathbf{B}\mathbf{N}\mathbf{N}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}}. \end{aligned} \tag{10}$$

As shown in (10) the transformation also affects the noise component, leading to colored noise at the output. This requires that some sort of prewhitening is applied prior to the DOA estimation. For prewhitening schemes, we refer to [19] for the matrix case and [20] for the tensor case.

The calculation of $\mathbf{B}$ will usually be an overdetermined one, since there will be more discrete angles in the set $\mathcal{S}$ than antennas in the virtual array. This results in an imperfect transformation, a measure of this imperfection is the transformation error given by

$$\epsilon(\mathcal{S}) = \frac{\left\|\bar{\mathbf{A}}_{\mathcal{S}} - \mathbf{B}\mathbf{A}_{\mathcal{S}}\right\|_{\mathrm{F}}}{\left\|\bar{\mathbf{A}}_{\mathcal{S}}\right\|_{\mathrm{F}}} \in \mathbb{R}^{+}. \tag{11}$$

Larger sectors will lead to larger transformation errors, and while it is possible to keep the transformation error as low as desired by keeping the sector sizes small this may lead to further problems such as demanding a very large number of estimations to be performed, one for each sector. It is possible to increase the number of antennas at the virtual array to obtain a smaller transformation error, this, however, will lead to an ill conditioned transformation matrix and to a large bias in the final DOA estimations. The number of antennas in the virtual array is usually chosen as to be equal or smaller then the number of antennas in the real array.

4 PROPOSED APPROACH

In this section a signal adaptive approach for array interpolation is shown and detailed in Subsections 4.1, 4.2 and 4.3.

4.1 Power Scanning and Sector Selection

Since the array response needs to be known to construct $\mathbf{B}$, such knowledge can be used to detect angular regions where significant power is received. This estimation can be done with the conventional beamformer [21], yielding the normalized power response

$$P(\theta) = \frac{\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}_{\mathbf{XX}}\mathbf{a}(\theta)}{\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{a}(\theta)} \in \mathbb{R}, \tag{12}$$

where $\hat{\mathbf{R}}_{\mathbf{XX}} = \frac{\mathbf{X}\mathbf{X}^{\mathrm{H}}}{N}$ is the estimate of the signal covariance matrix. In real systems the result of (12) is discrete in θ and can be written as

$$P[z] = P(-90^{\circ} + (z \cdot \Delta)) = P(\theta), \tag{13}$$

with $\theta \in \mathcal{D}_{\Delta}$ where

$$\mathcal{D}_{\Delta} = \{-90^{\circ}, -90^{\circ} + \Delta, ..., 90^{\circ} - \Delta, 90^{\circ}\} \tag{14}$$

and Δ is the resolution of the azimuth angle of the power response (12).

The output of (12) is scanned for sectors, and for each sector the respective lower bound $l_k \in \mathcal{D}_{\Delta}$ and upper bound $u_k \in \mathcal{D}_{\Delta}$ are defined as shown in Figure 1. The threshold

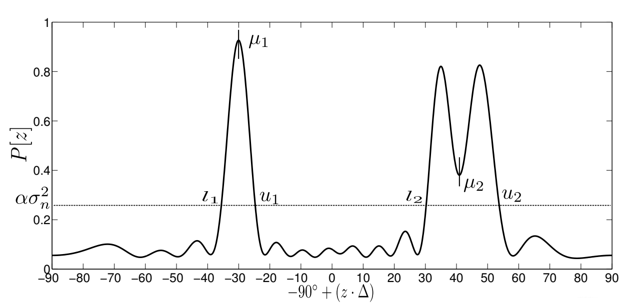

Figure 1: Selected sectors and respective bounds

that defines a sector and its bounds can be defined, for instance, by looking at the noise power. The noise floor can be set at $\alpha\sigma_n^2$, with $\alpha > 1$ being a sensitivity parameter. A large α means that only large sectors are detected but coming at the cost of discarding smaller sectors that are related to a signal component, while a small α means that smaller sectors are detected but at the cost of allowing noise to be mistakenly detected as a sector. If K sectors are detected, a detected sector with bounds $[l_k, u_k]$ is said to be centered at

$$\mu_k = \left\lceil \frac{|u_k - l_k|}{2} \right\rfloor_{\mathcal{D}_\Delta} \in \mathcal{D}_\Delta, \tag{15}$$

where $\lceil\cdot\rfloor_{\mathcal{D}_\Delta}$ is a rounding operation to the domain $\mathcal{D}_\Delta$. A weighting factor for each sector is calculated as

$$\xi_k = \frac{\sum_{z=l_k}^{u_k} P[z]}{\sum_{w=1}^{K}\sum_{z=l_w}^{u_w} P[z]} \in \mathbb{R}. \tag{16}$$

As mentioned in Section 2, classical array interpolation methods in the literature divide the array response into various sectors (sector-by-sector processing) in order to keep the error (11) small. However, in this work a single transform matrix is used based on a combination of the sectors detected in (12). In order to bound the error $\epsilon(\mathcal{S})$ a signal adaptive method for calculating the maximum transform size is used based on the weights calculated in (16). For a sector centered at μ_k, the discrete and countable set of angles used to transform this sector is given by

$$\mathcal{S}_k = \begin{cases} \left\{ \left\lceil \mu_k - \frac{\Xi\xi_k}{2} \right\rfloor_{\mathcal{D}_\Delta}, \left\lceil \mu_k - \frac{\Xi\xi_k}{2} + \Delta \right\rfloor_{\mathcal{D}_\Delta}, ..., \left\lceil \mu_k + \frac{\Xi\xi_k}{2} \right\rfloor_{\mathcal{D}_\Delta} \right\} & , \Xi\xi_k < |u_k - l_k| \\ \{l_k, l_k + \Delta, ..., u_k\}, & \Xi\xi_k \geq |u_k - l_k| \end{cases} \tag{17}$$

and

$$\mathcal{S}_k \cap \mathcal{S}_{\bar{k}} = \emptyset \; \forall \; k, \bar{k} \in \{1, ..., K\} \text{ and } \mathrm{k} \neq \bar{\mathrm{k}}, \tag{18}$$

where $\Xi \in \mathbb{R}^+$ is the total transform size in degrees of all sectors. The combined sector is given by

$$\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2 \cup ... \cup \mathcal{S}_K. \tag{19}$$

Thus, $\mathcal{S}$ has a better resolution for the sectors $\mathcal{S}_k$ where more power is present (weighted by ξ_k), i.e the transformation of the combined sector $\mathcal{S}$ is weighted towards the sectors $\mathcal{S}_k$ that include more signal power.

As the problem of obtaining the transform matrix $\mathbf{B}$ is equivalent to solving a highly overdetermined system we have

$$|\mathcal{S}| \to \infty \Longleftrightarrow \epsilon(\mathcal{S}) \to \infty. \tag{20}$$

Thus, transforming the entire detected sectors may result in a very high transformation error introducing a very large bias into the final DOA estimates. To address this problem an upper bound to the transform error $\epsilon_{\max}$ needs to be defined and a search can be performed to find the maximum transform size covering the detected sectors that is still within the error upper bound. The problem of finding the maximum Ξ with respect to $\epsilon_{\max}$ can be written as the optimization problem

$$\max_{\Xi} \; \epsilon(\mathcal{S}) \tag{21}$$

$$\text{subject to } \epsilon(\mathcal{S}) \leq \epsilon_{\max} \tag{22}$$

$$\Xi \leq \Xi_{\max} = \sum_{k=1}^{K} |u_k - l_k| \tag{23}$$

$$\Xi \geq \Xi_{\min} = M\Delta. \tag{24}$$

The problem in (21,22,23,24) can efficiently be solved using a bisection search method since, once all μ_k have been defined, the error function increases monotonically for $\Xi > \Xi_{\min}$, as illustrated in Figure 2. $\epsilon(\mathcal{S})$ is greatly affected if the calculation of $\mathbf{B}$ is either a heavily overdetermined or an underdetermined system. Therefore, $\Xi_{\min}$ is defined to ensure monotonicity of the problem given in (21).

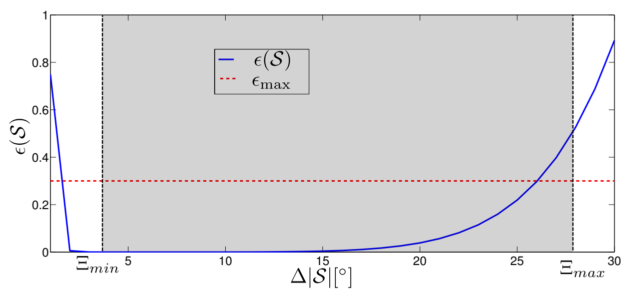

Figure 2: Transformation error with respect to combined sector size

4.2 Data Transformation

Once (21) has been solved and $\mathbf{B}$ has been calculated according to (6) the transformed covariance including FBA and SPS is obtained by [7]

$$\bar{\mathbf{R}}_{\mathbf{XX}} = \frac{1}{2L}\sum_{l=1}^{L} \mathbf{J}_l^{\mathrm{T}}(\mathbf{B}\hat{\mathbf{R}}_{\mathbf{XX}}\mathbf{B}^{\mathrm{H}} + \mathbf{Q}\mathbf{B}\hat{\mathbf{R}}_{\mathbf{XX}}^{*}\mathbf{B}^{\mathrm{H}}\mathbf{Q})\mathbf{J}_l, \tag{25}$$

where $(\cdot)^*$ stands for the complex conjugation, $\mathbf{Q}$ is a matrix containing ones in its anti-diagonal and zeros elsewhere, $\mathbf{J}_l$ is an appropriate selection matrix responsible for selecting the sub-arrays to be averaged, and L is the total number of sub-arrays employed. While L can be chosen *a priori* it can also be adaptively chosen as to minimized the loss of effective array aperture while achieving a good estimate of d. This can be done using a model order estimation method.

Model order selection is selecting the optimal trade-off between model resolution and its statistical reliability. In the specific case of this work, model order selection is mostly employed to select the eigenvectors of the signal covariance matrix that account for most of its power, each of these eigenvectors in turn represent the statistics of a received signal. Therefore, in this work, model order selection is mostly used to estimate the number of signals received at the antenna array. This is done by analyzing the profile of the eigenvalues of the signal covariance matrix and looking for a big gap that should separate the eigenvalues related to the signal subspace from the ones related to the noise subspace. If the signals are highly correlated a single eigenvalue can be related to two or more signals, which in turn will lead to a biased estimation. For this reason the aforementioned FBA and SPS must be applied in such cases.

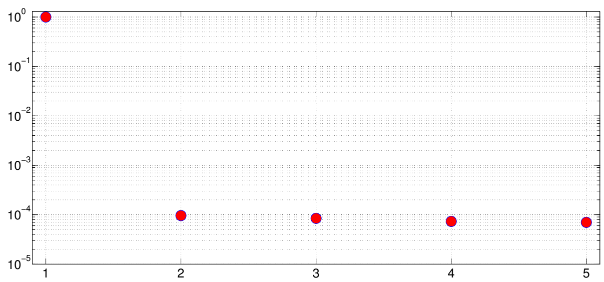

Figure 3: Eigenvalue profile before FBA-SPS: eigenvalue index versus eingenvalue

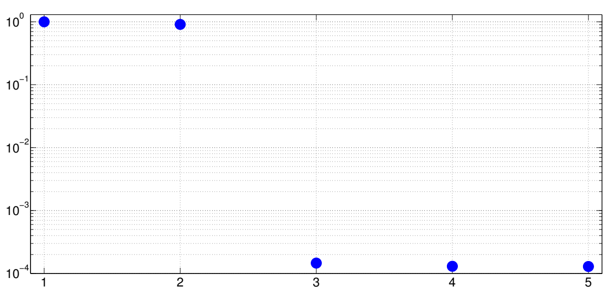

Figure 4: Eigenvalue profile after FBA-SPS: eigenvalue index versus eingenvalue

Figures 3 and 4 presents the example of an eigenvalue profile of two incoming signals before and after FBA and SPS have been applied, respectively. The SNR in this case is 30 dB and eight antennas are used. Note the great effect that FBA and SPS have on the eigenvalues, effectively two large eigenvalues can be seen after FBA and SPS whereas before there is only a single large eigenvalue. Model order selection schemes such as the AIC and the MDL methods [22] and more recently the RADOI [23] are capable of properly detection the number of signals received in the eigenvalue profile shown in Figure 4. For multidimensional problems more accurate methods such as [24], [25] can be used.

We use as a model order estimation method $\mathrm{MOE}(\bar{\mathbf{R}}_{\mathbf{XX}}(L)) = \hat{d}$ the RADOI [23]. Therefore we have to solve the problem

$$(L, \hat{d}) = \arg\min_{L} \max_{\hat{d}} \left\{ \mathrm{MOE}(\bar{\mathbf{R}}_{\mathbf{XX}}(L)) \right\}. \tag{26}$$

It is important to notice that the estimated number of impinging signals $\hat{d}$ can be different from the number of sectors detected in (12). Each of the detected sectors $\mathcal{S}_k$ can be formed by a set of nearly coherent signals that now can be efficiently separated with (26) allowing the application of a high resolution DOA estimation method to jointly estimate the parameters of all the detected signals.

4.3 GEVD and ESPRIT

Once the number of signals has been estimated and with the FBA-SPS covariance matrix at hand a joint estimation of the DOAs of all the incoming signals can be performed. After FBA and SPS DOA estimation can be done with any DOA estimation method. For this work we choose the ESPRIT method since it is a closed form algorithm that can be very easily extended to multidimensional scenarios. It is important to highlight that the the current state of array interpolation in the literature [15] states that ESPRIT cannot be employed with a transformation matrix calculated as shown previously in this work. Thus, although this section does not present any new method for DOA estimation it is still novel to apply ESPRIT to a transformation matrix applied directly to the signal covariance matrix.

The ESPRIT parameter estimation technique is based on subspace decomposition. Matrix subspace decomposition is usually done by applying the Singular Value Decomposition (SVD). The SVD of the matrix $\mathbf{X} \in \mathbb{C}^{M \times N}$ is given by

$$\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^{\mathrm{H}}, \tag{27}$$

where $\mathbf{U} \in \mathbb{C}^{M \times M}$ and $\mathbf{V}^{N \times N}$ are unitary matrices called the left-singular vectors and right-singular vectors of $\mathbf{X}$ and $\mathbf{\Lambda} \in \mathbb{C}^{M \times N}$ is pseudo diagonal matrix containing the singular values of $\mathbf{X}$. The signal subspace $\mathbf{E}_S \in \mathbb{C}^{M \times \hat{d}}$ of $\mathbf{X}$ can be constructed by selecting only the singular vectors related to the $\hat{d}$ largest singular values, the remaining singular vectors form the noise subspace $\mathbf{E}_N \in \mathbb{C}^{M \times M - \hat{d}}$ of $\mathbf{X}$.

Equivalently eigenvalue decomposition can be applied on the auto covariance matrix $\hat{\mathbf{R}}_{\mathbf{XX}}$ of $\mathbf{X}$ spanning the same subspace

$$\hat{\mathbf{R}}_{\mathbf{XX}} = \mathbf{E}\mathbf{\Sigma}\mathbf{E}^{-1}, \tag{28}$$

where $\mathbf{E} \in \mathbb{C}^{M \times M}$ and $\mathbf{\Sigma} \in \mathbb{C}^{M \times M}$ contains the eigenvectors and eigenvalues of $\mathbf{R}_{\mathbf{XX}}$. The eigenvectors related to the $\hat{d}$ largest eigenvalues span the same signal subspace $\mathbf{E}_S$ of the single value decomposition. The same holds for the noise subspace of the EVD and left singular vectors of the SVD, $\mathbf{E}_N$.

This classic eigendecomposition is suitable when the noise received at the antenna array is spatially white, since, in our case, we apply a transformation to the data, even if the received noise was originally white it turns into colored noise. To deal with colored noise the generalized eigenvalue decomposition (GEVD) can be used to take the noise correlation into account, the GEVD of the matrix pair $\bar{\mathbf{R}}_{\mathbf{XX}}, \bar{\mathbf{R}}_{\mathbf{NN}}$ is given by

$$\bar{\mathbf{R}}_{\mathbf{XX}}\bar{\mathbf{\Gamma}} = \bar{\mathbf{R}}_{\mathbf{NN}}\bar{\mathbf{\Gamma}}\mathbf{\Lambda}, \qquad (29)$$

where $\bar{\mathbf{\Gamma}} \in \mathbb{C}^{M\times M}$ is a matrix containing the generalized eigenvectors and $\mathbf{\Lambda} \in \mathbb{R}^{M\times M}$ is a matrix containing the generalized eigenvalues in its diagonal and $\bar{\mathbf{R}}_{\mathbf{NN}}$ is obtained by replacing $\hat{\mathbf{R}}_{\mathbf{XX}}$ with an estimate of the noise covariance $\hat{\mathbf{R}}_{\mathbf{NN}}$ in (25). Notice that this decomposition is the same as the EVD (28) for $\bar{\mathbf{R}}_{\mathbf{NN}} = \mathbf{I}$. The subspace $\bar{\mathbf{\Gamma}}_S \in \mathbb{C}^{M\times\hat{d}}$ is formed selecting the generalized eigenvectors related to the $\hat{d}$ largest generalized eigenvalues. This subspace, however, does not span the same column subspace as the original steering matrix, and needs to be reprojected onto the original manifold subspace or dewhitened. This can be done by

$$\mathbf{\Gamma}_s = \bar{\mathbf{R}}_{\mathbf{NN}}\bar{\mathbf{\Gamma}}_s. \qquad (30)$$

With this subspace estimate at hand the Total Least Squares (TLS) ESPRIT [14] is applied. Two subsets of the signal subspace that are related trough the shift invariance property need to be selected. The choice depends on with parameter is to be estimated and are directly dependent on the way the data has been organized. Figure 5 shows an example for a rectangular array with dual polarization, in the figure black and gray circles represent antennas with different polarizations. The TLS-ESPRIT algorithm presented does not depend on subset selection, and will be presented in a general way.

Let $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ represent the subspace subsets selected in as previously mentioned. A matrix $\mathbf{\Gamma}_{1,2}$ is constructed as

$$\mathbf{\Gamma}_{1,2} = \begin{bmatrix} \mathbf{\Gamma}_1^{\mathrm{H}} \\ \mathbf{\Gamma}_2^{\mathrm{H}} \end{bmatrix} [\mathbf{\Gamma}_1\mathbf{\Gamma}_2], \qquad (31)$$

by performing and eigendecomposition of $\mathbf{\Gamma}_{1,2}$ and ordering its eigenvalues in the decreasing order and its eigenvectors accordingly the eigenvector matrix $\mathbf{V}$ can be divided into blocks as

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{1,1} & \mathbf{V}_{1,2} \\ \mathbf{V}_{2,1} & \mathbf{V}_{2,2} \end{bmatrix}. \qquad (32)$$

Finally, the parameters can be obtained by finding the eigenvalues of

$$\Phi = \mathrm{eig}\left(-\frac{\mathbf{V}_{1,2}}{\mathbf{V}_{2,2}}\right). \qquad (33)$$

The parameters in Φ can represent a phase delay respective to a DOA if DOAs are being estimated, or can represent the ratio of the strength at which a signal appears in the different polarizations, respectively.

For multidimensional arrays another option is to employ methods based on the PARAFAC decomposition such as [26], [27] instead of the ESPRIT.

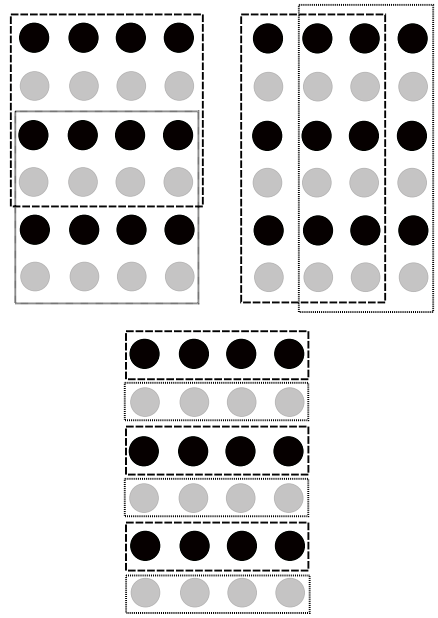

Figure 5: Example of select subsets

Finally, once the first set of estimates has been obtained the Vandermonde Invariance Transformation (VIT) can be applied, this transformation aims to shape the noise alway from the regions where the signal is arriving in order to obtain improved estimates [17] . The VIT is an array interpolation approach that transforms a Vandermonde system with a response linear with respect to the angle of arrival into a Vandermonde system with a highly nonlinear response with respect to the angle of arrival. The VIT promotes a nonlinear transformation with respect to the selected spatial frequency $\mu(\phi)$. Let $\mathbf{u} = [1, \mathrm{e}^{\mathrm{j}\mu(\phi)}, \ldots, \mathrm{e}^{\mathrm{j}\mu(\phi)(\mathrm{M}-1)}]$, the VIT performs the following transformation

$$\bar{\mathbf{u}}^{(VIT)} = \mathbf{T}(\phi)\mathbf{u} = (\frac{\mathrm{e}^{\mathrm{j}\mu(\phi)} - r}{1-r}) \begin{pmatrix} 1 \\ e^{\mathrm{j}\nu(\phi)} \\ \vdots \\ e^{\mathrm{j}\nu(\phi)(M-1)} \end{pmatrix}, \qquad (34)$$

r and ν are design parameters that can be chosen considering a compromise between the level of noise suppression desired around μ and the linearity of the output.

The VIT can be used to apply a phase attenuation to the dataset, which in turn shapes the noise, reducing the power of the noise in the region near $\mu(\phi)$ and increasing it over its vicinity. Thus, the VIT needs to be applied angle wise, i.e, a set of initial estimates of the angles ϕ is used to calculate a VIT centered over the given angles, and a second estimate is performed. This second estimation yields and offset ϕ_{offset} with respect to the original ϕ, giving the final estimation $\phi_{\mathrm{VIT}} = \phi + \phi_{\mathrm{offset}}$. Due to the mentioned noise shaping, this final estimation offers increased precision, but comes at the cost of transforming the dataset and applying

the chosen DOA estimation method $\hat{d}$ times.
The VIT can be interpreted as a zoom, similar to an optical zoom, with the first estimates a zoom can be used on the regions of the manifold where signal has been estimated to arrive and the region can be inspected with the zoom effect to detect any imprecisions from the first estimate. The increased performance comes at the cost of $\hat{d}$ extra DOA estimations.

5 NUMERICAL SIMULATIONS AND DISCUSSION

To study the performance of the proposed method a set of numerical simulations is performed. The performance of the proposed method is assessed in the presence of spatially white Gaussian noise and errors in the known array response. The known array response used in the simulations is constructed by randomly displacing the elements of a Uniform Linear Array (ULA) composed of $M = 8$ antennas with element spacing of $\frac{\lambda}{2}$ to a point on a circle with its center on the original antenna position and radius $\frac{0.1\lambda}{2}$, where λ is the wavelength of the carrier frequency of the signal. Figure 6 shows a graphical representation of how the antennas are displaced. While physical displacement is used in this work the same method can be used to deal with non-linearity in the antenna array with respect to the DOAs of the received signals or with imperfect responses of individual antennas in the array. For each simulation run a different point is randomly chosen within the displacement circle as to avoid a displacement where all sensors are displaced in similar directions, resulting in a small relative displacement.
$\hat{\mathbf{R}}_{\mathbf{XX}}$ is obtained using $N = 200$ snapshots and 2e assume three signals impinging from $\theta_1 = 45°$, $\theta_2 = 38°$, and $\theta_1 = 15°$. The Root Mean Squared Error (RMSE) is calculated with respect to 1000 Mont Carlo simulations and is given by

$$\text{RMSE} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left((\hat{\theta}_{1,k} - \theta_1)^2 + (\hat{\theta}_{2,k} - \theta_2)^2 + (\hat{\theta}_{3,k} - \theta_3)^2\right)}, \tag{35}$$

where $\hat{\theta}_i$ is the estimate of θ_i. The Signal to Noise Ratio (SNR) is defined as

$$\text{SNR} = \frac{\sigma_1^2}{\sigma_\text{n}^2} = \frac{\sigma_2^2}{\sigma_\text{n}^2} = \frac{\sigma_3^2}{\sigma_\text{n}^2}.$$

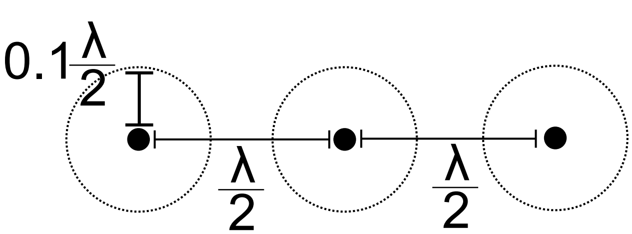

Figure 6: Graphical example of antenna displacement

To assess the performance of the proposed method under demanding conditions the set of transmitted signals is highly correlated. The wavefronts impinging from θ_1 and θ_2 are correlated with correlation coefficient $\rho = 1$ and correlated to the wavefront impinging from θ_3 with $\rho = 0.8$. The two parameters that fully define the proposed approach given in in Subsections 4.1, 4.2 and 4.3 are set to $\alpha = 1.2$ and $\epsilon_{\max} = 10^{-3}$.

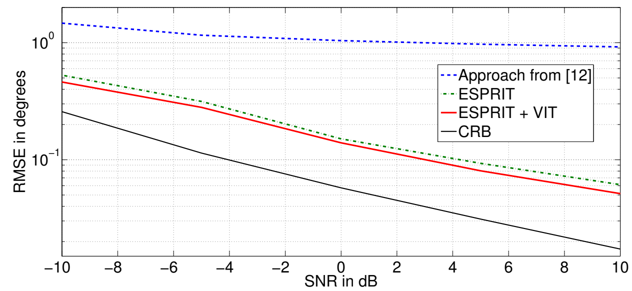

Figure 7: RMSE for [12], ESPRIT and ESPRIT+VIT

In [12] a sector-by-sector processing with out-of-sector response filtering and applying MUSIC is proposed. We use the same approach for the calculation of the transformation matrices as given in [12] while additionally applying FBA and SPS as well as using Root-MUSIC instead of MUSIC. Furthermore, we assumed, for the simulations of the approach given in [12] that the model order is perfectly known. Figure 7 shows that the estimates provided by the approach of [12] with Root-MUSIC quickly reach a plateau for this demanding signal scenario as the DOA estimation bias is dominated by large transformation errors for each sector. On the other hand, the approach proposed in this work provides improved accuracy with increasing SNR since the size of the combined sector also decreases, resulting in a much smaller transformation error. The proposed approach is still not capable of reaching the CRB due to the application of SPS, which effectively decreases array aperture, however, L being chosen according to (26).

6 CONCLUSION

In this work a novel, low complexity, signal adaptive method for DOA estimation with interpolated arrays is presented. The received signal is used to obtain a single transform matrix instead of the traditional sector-by-sector processing. The closed form ESPRIT together with the VIT is applied without the need of explicitly prewithening the transformed covariance and the joint estimation of all waveforms avoids the problem of selecting different sector estimates (double estimates or "ghost" signals). The proposed approach can easily be extended for multidimensional signal processing for arrays with arbitrary array geometry. The proposed approach achieves improved accuracy in DOA estimation with respect to state of the art methods and provides robustness to errors in the array response model. Especially, spoofing and multipath signals can be detected and mitigated reliably together with adapted RAIM algorithms. Furthermore, precise attitude estimation for antenna arrays is enable by highly accurate DOA estimated even when coherent multipath or spoofing signals are present.

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