

# Sensor Localization Via Diversely Polarized Antennas

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**Abstract**—Wireless sensor networks (WSNs) are currently employed in a vast number of different applications ranging from home automation and health care to military systems. Although their application may vary greatly, WSNs share a common set of characteristics such as a limited energy supply and simple hardware. A common issue related with the application of WSNs is sensor localization, for some types of applications it is important that the sensors know the relative or absolute position of other sensors in the network, such as surveillance of monitoring networks. If sensors are randomly placed they may resort a wide range of methods such as Global Navigation Satellite Systems (GNSS) or received signal strength indicators (RSSI). In this work we present an alternative to relative sensor localization by employed a crossed dipole antenna in the reception and a known polarization in the transmission. The accuracy of the proposed methods is measured through numerical simulations and results are presented.

**Index Terms**—Wireless Sensor Networks, Sensor Localization

## I. INTRODUCTION

In recent years wireless sensor networks (WSNs) have been employed in a large number of applications. Their usage ranges from military applications, such battlefield surveillance and targeting, to health applications, such as automating drug applications in hospitals [1]. Some of these applications, battlefield surveillance for instance, require the positions of the sensors to be known across the network either in a relative or absolute manner. Sensor localization also allows Cooperative MIMO [2], [3], [4] to be used in WSNs, providing numerous benefits. Since sensors can be deployed in a random manner an automatic method of sensor location is required.

Absolute positioning can be known by equipping each node with a Global Navigation Satellite System (GNSS) module such as a Global Positioning System (GPS) module. However, most WSNs are composed of highly simple hardware and also posses a highly limited energy budget, thus making this alternative impractical or even impossible. Employing GNSS also results in a network that is no longer self contained, as it relies on the presence of an external system to properly function.

A large number of alternatives have been proposed for relative sensor localization. One of the simplest ones is estimating the distance between nodes by measuring the power of a received signal, this approach requires a very precise model of signal attenuation which may be hard to obtain [5] and a fairly stable operation environment.

Another proposed method is by measuring the number of times a packet needs to be retransmitted in order to reach a given destination. The Radio Hop Count [6] is capable of providing a relative localization across the network without the presence of additional hardware and with a better precision than the RSSI method.

Some methods rely on analyzing the difference between the data measured by each sensor and trying to fit the measured data to a given positioning model. In [7] the sounds measured by a network of microphones are used in order to estimate the positions of the sensors in the environment.

The time of arrival (TOA) is another method used for estimating the distance between a pair of sensors. This can be done in two ways. Assuming perfect clock synchronization between two nodes a normal radio transmission can be scheduled, the receiving node can then measure the time difference between the scheduled transmission and the time when the signal was actually received. Another way of using the TOA is to used a secondary transmission with different propagation speed, such as a sound wave, and measure the time difference between the reception of the radio and the sound signals. For precise positioning this method requires either a very precise synchronization between nodes, which may be unachievable, or the presence of additional hardware such as an ultrasound transmitter and microphone.

The direction of arrival (DOA) is also an important alternative [8]. This method relies on measuring the angle at which a signal arrives at the sensor given a reference position. Once the DOAs of received signals are estimated an estimate of the location of transmitting sensors can be obtained. A large number of methods exist to precisely estimate the DOAs of received signals, however, most techniques rely on the presence of antenna arrays.

In this paper we present an alternative to sensor localization using DOA estimation. Instead of relying on antenna arrays we employ only a single crossed dipole antenna, extending the work seen in [9] to a simpler scenario.

The reminder of this paper is divided into five sections. In section II the basic assumptions are presented. In III the model assume for radio wave reception and propagation is presented. In section IV the proposed method is shown in detail. In section V numerical results are shown and analyzed. Finally, conclusions are drawn in section VI.

## II. PROBLEM DESCRIPTION

We assume a network formed consisting of  $K$  sensors randomly placed at points  $S_1, S_2, \dots, S_K$  where

$$S_1 = [x_1, y_1]. \quad (1)$$

We also assume the presence of a set of nodes  $S_i, \dots, S_j$  that possess prior information about their own localization in the network. The proposed method requires

$$|i, \dots, j| \geq 3. \quad (2)$$

It is also assumed that the orientation of all sensors is known with respect to a common reference, this can be done by employing a beacon transmitter or assuming an internal compass is present.

## III. SIGNAL MODEL

The electric field of a propagating wave can be presented as

$$\mathbf{E} = -E_x \mathbf{e}_x + E_y \mathbf{e}_y, \quad (3)$$

where  $E_x$  and  $E_y$  are the horizontal and vertical components of the electric field. These components define a polarization ellipse shown in Figure 1.

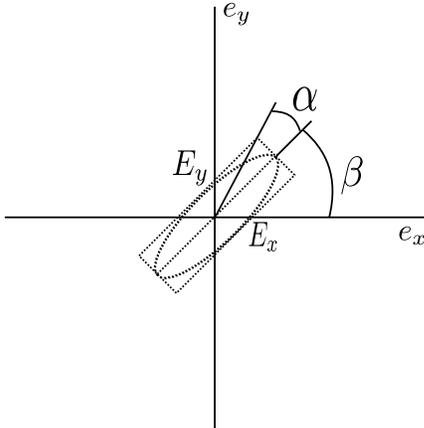


Figure 1: Polarization ellipse

Note that the electric field components can be written now in terms of the electric angles  $\alpha$  and  $\beta$  as

$$E_x = \mathbf{E} \cos(\gamma) \quad (4)$$

$$E_y = \mathbf{E} \sin(\gamma) e^{j\eta} \quad (5)$$

where

$$\cos(2\gamma) = \cos(2\alpha) \cos(2\beta) \quad (6)$$

$$\tan(\eta) = \tan(2\alpha) \csc(2\beta). \quad (7)$$

as shown in Figure 2.

Considering a wavefront impinging over a crossed dipole with components parallel to the  $x$ - and  $y$ -axis of the polarization ellipse the output at such dipoles will be proportional to the components  $E_x$  and  $E_y$  respectively and be written as

$$\mathbf{E} = (-E_x) \mathbf{e}_x + (E_y \cos(\theta)) \mathbf{e}_y \quad (8)$$

$$= \mathbf{E} (-\cos(\gamma) \mathbf{e}_x + \sin(\gamma) \cos(\theta) e^{j\eta} \mathbf{e}_y) \quad (9)$$

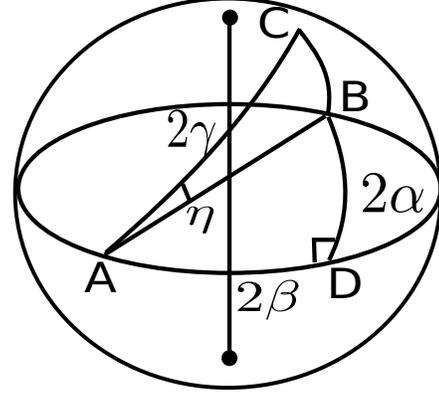


Figure 2: Poincaré sphere

where  $\theta$  is the angle of arrival of the received wave with respect to the  $x$ -axis.

The received signal can now be expressed in matrix form as

$$\mathbf{X} = \mathbf{u} \mathbf{s} + \mathbf{N} \quad (10)$$

where  $\mathbf{X} \in \mathbb{C}^{2 \times N}$  is the matrix containing the measured outputs at each of the dipoles,  $N$  is the number of measured snapshots,  $\mathbf{s} \in \mathbb{C}^{1 \times N}$  is the vector containing the original transmitted signal transmitted,  $\mathbf{N} \in \mathbb{C}^{2 \times N}$  is the matrix containing the Additive White Gaussian noise present at sampling and the polarization vector  $\mathbf{u} \in \mathbb{C}^{2 \times 1}$  is defined as

$$\mathbf{u} = \begin{bmatrix} -\cos(\gamma) \\ \sin(\gamma) \cos(\theta) e^{j\eta} \end{bmatrix} \quad (11)$$

according to (9). We assume for the remainder of this work that all the polarization components are known for at all receivers.

## IV. PROPOSED METHOD

In this section we present two different alternatives for DOA estimation using the electrical angle in Subsections IV-A and IV-B, we compare both alternatives in Subsection IV-C, and present a method for removing any ambiguities in DOA estimation and localization in Subsection IV-D.

### A. DOA Estimation Using Electrical Angle Ratio

The approach proposed in this work consists of analyzing the ratio between the outputs of the crossed dipole in order to estimate the direction of arrival of the received signal. Given the ratio

$$r = \frac{-\cos(\gamma)}{\sin(\gamma) \cos(\theta) e^{j\eta}}, \quad (12)$$

the angle  $\theta$  can be obtained by

$$\theta = \cos^{-1} \left( \frac{-\cos(\gamma)}{r \sin(\gamma) e^{j\eta}} \right). \quad (13)$$

One way to estimate the ratios is to simply average a large number of samples from both antennas in order to reduce the effects of the noise and obtain the ratio between the powers. It is important to highlight at this point that although there is a need for a crossed dipole with two independent outputs to

be present it is not necessary that both outputs are connect to individual receiver radios. A single symbol can be measured at both outputs by dividing the symbol duration, assuming the transmitted energy is constant over the entire symbol duration.

### B. DOA Estimation Using Electrical Angle ESPRIT

A more precise approach is to employ the ESPRIT [10] algorithm in order to obtain the ratio between the received symbols. The advantage of the ESPRIT algorithm is that it is capable of dealing much more efficiently with noise since it relies on an eigendecomposition that separates noise and signal subspaces. The first step of the ESPRIT algorithm used in this work is to form the covariance matrix  $\mathbf{R}_{\mathbf{X}\mathbf{X}} \in \mathbb{C}^{2 \times 2}$  of the received signal

$$\mathbf{R}_{\mathbf{X}\mathbf{X}} = \frac{\mathbf{X}\mathbf{X}^H}{N}, \quad (14)$$

where the operator  $(\cdot)^H$  stands for the conjugate transposition. The second step is to obtain the eigendecomposition of  $\mathbf{R}_{\mathbf{X}\mathbf{X}}$

$$\mathbf{R}_{\mathbf{X}\mathbf{X}} = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^{-1}. \quad (15)$$

Since only one sensor should be transmitted while the others are receiving or sleeping one can construct the so called signal subspace  $\mathbf{E}_s \in \mathbb{C}^{2 \times 1}$  by selecting the eigenvector related to the largest eigenvalue. An estimate of the ratios can now be easily obtained by

$$r = \left| \frac{\mathbf{E}_s[2]}{\mathbf{E}_s[1]} \right|. \quad (16)$$

### C. Comparison between Ratio and ESPRIT solutions

Although the ESPRIT method is capable of obtaining much more precise estimates as shown in Figure 3 it generates a larger computational load. This load is, however, justified as a small error in the estimated ratio may lead to a large error in the estimation of the DOA. This computationally demanding step is also only necessary once every time the topology of the network changes, thus, the ratio at which sensors change location needs to be taken into account when choosing the estimation method.

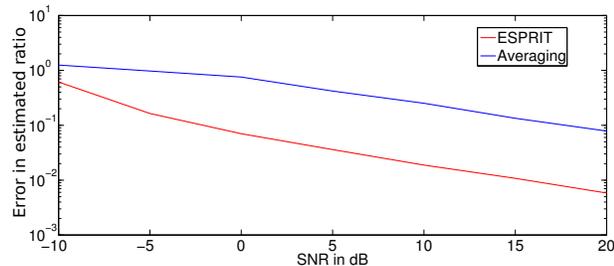


Figure 3: Comparison between ESPRIT and averaging samples

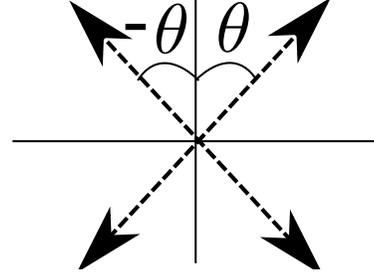


Figure 4: Depiction of possible ambiguity in signal propagation direction

### D. Removing DOA Ambiguities

It is important to notice that the DOA is given with respect to the reference of the  $x$ -axis and cannot distinguish from with direction, front or rear, the signal is arriving. Figure 4 displays this phenomenon.

It is also possible to notice from Figure 4 that  $\theta$  ranges from  $[-\pi, \pi]$  and thus it is also not possible to tell from with sector is the signal actually arriving since the only information that the system has is  $\cos(\theta)$ . The end result is that each sensor possesses two estimated lines in the ground plane where the transmitting node may be located. However, by acquiring a set of line estimates it is possible to obtain a single estimate of the transmitting sensor localization. Figure 5 shows an example of imprecise estimates from three receiving nodes being used to estimate the position of the transmitter node. The problem is reduced to the least squares problem of finding the point with minimum distance from any of the possible combination of line estimates.

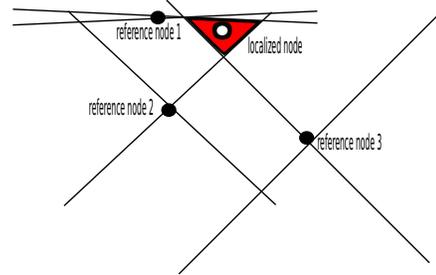


Figure 5: Sensor triangulation example using only the DOAs of the reference nodes

By writing the representing the line estimates as line equations of the type  $Ax + By + C = 0$  in the sensor coordinate system, the estimate of the sensor position can be found by solving

$$\{\hat{x}_0, \hat{y}_0\} = \min_p \frac{|A_{p1}x_0 + B_{p1}y_0 + C_{p1}|}{\sqrt{A_{p1}^2 + B_{p1}^2}} + \frac{|A_{p2}x_0 + B_{p2}y_0 + C_{p2}|}{\sqrt{A_{p2}^2 + B_{p2}^2}} + \dots, \quad (17)$$

where  $p$  is an index set containing the possible combinations of estimated lines. While more than three sensors can be used to obtain increased accuracy it also results in a higher computational load involved in the calculation of the minima. Once the lines are choose the final location estimate  $S_0 = [x_0, y_0]$  can be found by taking the derivative of the above with respect to  $x_0$  and  $y_0$  and finding the point where it is equal to zero.

Furthermore, this technique may be used in conjunction with other localization methods such as the RSSI. A set of candidate locations can be selected as shown in (14) and the RSSI information can be used to choose the candidate that best fits the RSSI information.

The whole localization of the entire network can be achieved as the individual sensor localizations are spread across the network.

## V. RESULTS AND DISCUSSION

The first result analyzed is how the precision of the estimated location is affected by the SNR of the transmitted signal. For this simulation the transmitting node is centered around three receiving nodes. The distance between the transmitting node and the receiving nodes is 100 m. 50 snapshots are used for the ratio estimation and both the averaging and the ESPRIT methods are compared. The scenario is detailed in Figure 6.

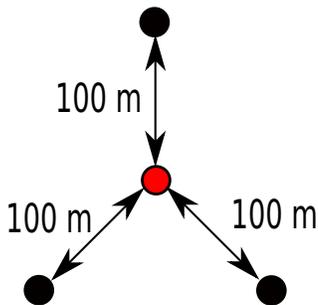


Figure 6: Illustration of first simulation scenario

The results shown in Figure 7 show how the error is affected by both the SNR and the selection of estimation method. Even for negative SNRs the ESPRIT method is capable of keeping the estimation error below 1 m. However, this scenario offers the advantage of a transmitting node centered in relation to the receiving nodes, thus, errors in angle estimation at each receiving node are more likely to end up compensating for each other.

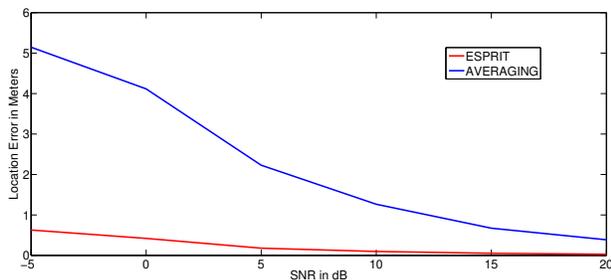


Figure 7: Location error for the first scenario

To better analyze how the proposed method behaves in a harsher scenario, we place the localized sensor non centered with respect to the reference sensors. Figure 8 presents graphically how the sensors were placed for the simulation.

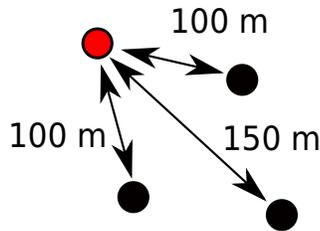


Figure 8: Illustration of second simulation scenario

Figure 9 shows how the error is affected not only by the SNR but also by the position of the receiving sensors. Since all sensors are located within a single sector with respect to the transmitting sensor, the angle estimation errors result in a larger positioning error, since the estimated intersection or near intersection of the lines is more likely to be displaced by larger distances.

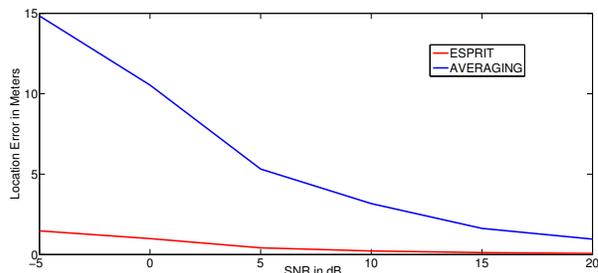


Figure 9: Location error for the second scenario

Finally to analyze the behavior of the proposed technique in an entire network environment and area of  $1 \times 1$  km is filled with enough sensors to guarantee that every sensor has a neighbor within 150 m with probability 0.98. The number of nodes can be calculated using the formula

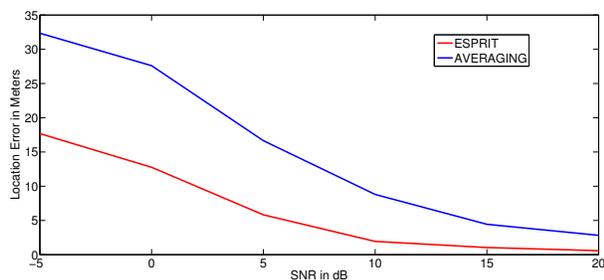
$$P = (1 - e^{-d\pi r^2})^n, \quad (18)$$

where  $d$  is the sensor density in the area,  $r$  is the range at which the probability is to be calculated and  $n$  is the number of sensors deployed. In this case the necessary number of sensors is 130 according to (18).

Figure 10 shows the location error averaged across the entire network. Since the location errors propagate the end result is higher location errors than when only a single estimation is considered. However, for high SNRs it is possible to notice that estimations within a 5 m error margin are achievable with the averaging method and with a 2 m margin when employing the ESPRIT method.

## VI. CONCLUSION

In this work we have presented a direction of arrival based localization method. The direction of arrival estimation is done by employing a crossed dipole antenna and does not rely on the presence of an antenna array. With proper knowledge of the polarization used for transmission the direction of arrival can be estimated by measuring the ratio of the power



**Figure 10:** Location error for full network

received across both dipoles. Ratio estimation can be done simply by averaging the received samples or by applying an ESPRIT type method for estimation, the latter offer increased precision at the cost of increased computational complexity. A simple minimization procedure is proposed in order to obtain the position estimation based on at least three independent direction of arrival estimations. Finally the proposed method is analyzed by means of numerical simulations and the precision of the proposed method is shown to depend on both SNR and receiving sensor localization. Results are shown to be within 2 m of precision for high SNRs in moderately dense networks.

#### ACKNOWLEDGMENTS

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