Direction of Arrival Estimation Performance for Compact Antenna Arrays with Adjustable Size

S. Caizzone *, W. Elmarissi *, M.A.M. Marinho†‡, F. Antreich §
*Inst. of Communications and Navigation, German Aerospace Center (DLR),Wessling, Germany
†Dep. of Electrical Engineering, University of Brasília (UnB), Brasília, Distrito Federal, Brazil
‡Halmstad University, School of Information Technology, Halmstad, Sweden
§Dep. of Teleinformatics Engineering, Federal University of Ceara (UFC), Fortaleza, Brazil
Email: stefano.caizzone@dlr.de

***Abstract*—The quest for compact antenna arrays able to perform robust beamforming and high resolution direction of arrival (DOA) estimation is pushing the antenna array dimensions to progressively shrink, with effects in terms of reduced performance not only for the antenna but also for beamforming and DOA estimation algorithms, for which their assumptions about the antenna properties do not hold anymore. This work shows the design and development of an antenna array with adjustable mutual distance between the single elements: such setup will allow to scientifically analyse the effects that progressive miniaturization, i.e. progressively smaller mutual distances between the antennas, have on the DOA estimation algorithms, as well as show the improvements obtained by using array interpolation methods, i.e. techniques able to create a virtual array response out of the actual array one, such as to comply with the algorithms' requirements on the antenna response.**

***Index Terms*—GNSS, array, miniaturization, DRA, array interpolation**

I. Introduction

In the last years, research on miniaturized antenna arrays has gained in scientific importance as well as in industrial interest. In the field of satellite navigation, for instance, antenna arrays are being investigated as powerful countermeasures to jamming and spoofing [1], [2], [3]. The dimensions of the array however must be strongly diminished from the canonical ones (having half wavelength ($\lambda/2$) separation between elements), in order to comply with installation requirements for mobile applications [4], [5]. Such requirement poses strong challenges in the antenna design, because the miniaturization of single elements and of the antenna array will have to be achieved while at the same time counteracting the side effects of mutual coupling and pattern distortion.

On the other hand, miniaturization has also an impact on the algorithm performance: several direction of arrival (DOA) estimation methods, e.g. such as Root MUSIC [6] and unitary ESPRIT [7], rely on a Vandermonde or a centro-hermitian array response. Furthermore, in order to separate highly correlated signals in the eigenspace pre-processing techniques such as Forward Backward Averaging (FBA) [8] and Spatial Smoothing (SPS) [9] have to be applied and they also demand a Vandermonde or a centro-hermitian array response. Such strong assumptions on the actual array response do not strictly hold anymore in case of antenna pattern distortion as well as mutual coupling between the antennas arising from the miniaturization, thus progressively diminishing the performance of the algorithms and finally of the whole beamforming system. The aforementioned methods however have benefits with respect to methods which do not demand a specific array geometry or array response such as iterative maximum likelihood DOA estimation using the Expectation Maximization (EM) algorithm [10] or classical MUSIC [11], thanks to the lower computational load required for their execution.

In order to guarantee a Vandermonde or a centro-hermitian array response within detected sectors of the field of view of the array, so called linear and nonlinear array interpolation methods can be applied, which conceptually transform the measured antenna array response into a virtual one having the characteristics required by the algorithms [12], [13]. In general it is quite complicated to practically evaluate the goodness of such techniques along with miniaturization of antenna arrays: this could be accomplished by manufacturing different arrays, composed of the same number and type of elements, but having different mutual distances between them. However, such approach will introduce additional error terms and uncertainties, due to differences in the materials samples, manufacturing tolerances and so on, finally invalidating the possibility to perform a fair comparison.

This work will therefore show the design of a linear array of 6 miniaturized dielectric resonator antennas (DRAs), deployed over a carrier structure able to be adjusted, i.e. to change the mutual distance among the single antennas.

Such design enables us to analyze the response of "different" arrays, i.e. arrays having different mutual distances between the elements, without having to cope with differences in the manufacturing and tolerances for the different arrays, exactly because the antenna elements are always the same and so all changes experienced in the algorithms performance can be ascribed to different array geometries and not to manufacturing uncertainties.

The data for two different arrays, having mutual distance between the elements of respectively 0.4 and 0.2 λ, will be analyzed as a first step and it will be shown how, for the more compact arrays, the DOA estimation algorithm under consideration (ESPRIT) will produce an higher error, due to the divergence of the antenna characteristics from the assumptions. It will be further shown how, thanks to the array interpolation methods, an improved performance can be recovered also in this case.

II. Array Interpolation Algorithm

We consider a set of d wavefronts impinging onto an antenna array composed of M antenna elements. The analysis done in the present work is very general and can be applied to different signals. The received baseband signal can be expressed in matrix form as

$$\mathbf{X} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{N} \in \mathbb{C}^{M\times N}, \tag{1}$$

where $\boldsymbol{S} \in \mathbb{C}^{d\times N}$ is the matrix containing the N symbols transmitted by each of the d sources, $\boldsymbol{N} \in \mathbb{C}^{M\times N}$ is the noise matrix with its entries drawn from $\mathcal{CN}(0, \sigma_n^2)$, and

$$\boldsymbol{A} = [\boldsymbol{a}(\phi_1), \boldsymbol{a}(\phi_2), ..., \boldsymbol{a}(\phi_d)] \in \mathbb{C}^{M\times d}, \tag{2}$$

where ϕ_i is the azimuth angle of the $i-$th signal and $\boldsymbol{a}(\phi_i) \in \mathbb{C}^{M\times 1}$ is the antenna array response for the given azimuth (in this work, for simplicity sake, a constant elevation angle $\theta = 80°$ will be assumed).

Array interpolation can be performed using a linear approach. Linear approaches can achieve a high accuracy in DOA estimation if the real array response is somewhat similar to the desired array response. Linear array interpolation consists of finding a transformation matrix $\boldsymbol{B}$ that transforms the real array response $\boldsymbol{A}_{\mathcal{S}}$ for a given countable and discrete set of angles $\mathcal{S}$, called a sector, into the desired array response $\bar{\boldsymbol{A}}_{\mathcal{S}}$. Thus, the matrix $\boldsymbol{B}$ can be seen as the matrix that achieves the best transform between a set of vectors $\boldsymbol{A}_{\mathcal{S}}$ and $\bar{\boldsymbol{A}}_{\mathcal{S}}$. The simplest solution for obtaining $\boldsymbol{B}$ is a least squares fit via

$$\boldsymbol{B} = \bar{\boldsymbol{A}}_{\mathcal{S}}\boldsymbol{A}_{\mathcal{S}}^{\dagger} \in \mathbb{C}^{M\times M}, \tag{3}$$

where $(\boldsymbol{.})^{\dagger}$ stands for the Moore–Penrose pseudo-inverse. The transformation matrix $\boldsymbol{B}$, however, is usually not capable of transforming the response perfectly across the entire sector $\mathcal{S}$ except for the case where a large number of antenna elements is present or a very small sector is used. A signal adaptive sectors selection can be used to minimize DOA estimation bias, as shown in [12].

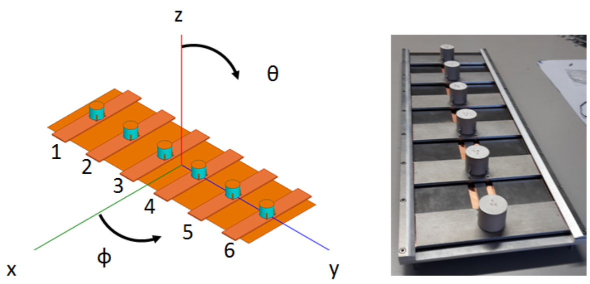
In cases where the real array response differs heavily from the desired array response, such as in miniaturized arrays of few elements, the linear interpolation method might not achieve sufficient DOA estimation accuracy. Furthermore, the performance of the linear interpolation approach relies heavily on the number of antennas available at the array. A larger number of antennas implicates a larger number of degrees of freedom for the least squares problem shown in (3). Therefore, a nonlinear approach can be useful for arrays with a limited number of antennas or for arrays where the real response differs heavily from the desired one. A candidate nonlinear regression approach is the multivariate adaptive regression splines (MARS) method, proposed in [15] as a non-parametric regression method that extended previous step-wise linear regression methods using splines. As a non-parametric regression method, MARS does not require any knowledge of the relationship between the predictor and predicted data. In [13] the MARS method, used in the following, was extended for the problem of antenna array interpolation.

III. Antenna Array Design

The designed array is composed of 6 cylindrical dielectric resonator antennas (for a review on DRAs, see for instance [14]), operating at GPS/Galileo L1 band (with central frequency of 1.57542 GHz). The single antennas are miniaturized through the use of a high dielectric constant (DK) ceramic material, having a DK~33 and hence allowing the antennas to be smaller than 2.5 cm x 2.5 cm x 2 cm, similarly as in [5].

They are fed through two metallic strips, conformal to the cylinder. By using an hybrid coupler below each antenna, providing a phase shift of 90° among the two feeding inputs, RHCP polarization is obtained.

The single antennas are fixed on small metallic slabs (Fig. 1), which can slide along guiding rails manufactured in a bigger metallic plate: once the wished mutual distance between the antennas has been achieved, the metallic slabs positions are fixed by tightening the guiding rails of the bigger metallic plate. The small dimensions of the single antennas are necessary to allow to place them quite close to each other, up to a minimum distance of 30 mm (~0.16 λ at L1 central frequency).



(a) (b)

Figure 1. 3D view of the 6 DRA array as simulated (a) and as manufactured (b): it is easy to notice how the small metallic slabs on which the single antennas are fixed can slide along the y-direction of the bigger metallic plate.

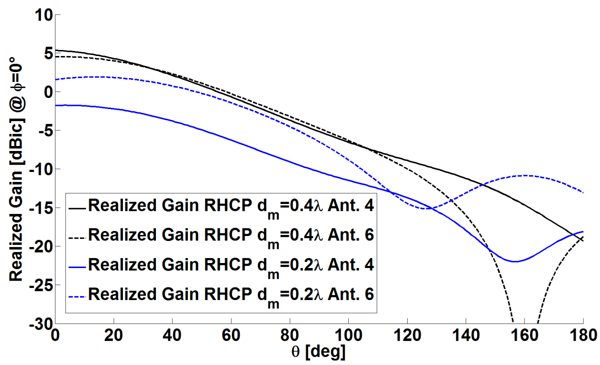


Figure 2. Antenna pattern of antennas no. 4 and 6 of the 6-element DRA array as obtained in HFSS (f=1575 MHz, $\phi = 0°$cut)

The antennas were simulated in HFSS for different mutual distances (0.2 and 0.4 λ): it is possible to clearly observe how the pattern of a central and an edge element are almost identical for $d_m = 0.4\lambda$, hence well matching the centro-hermitian condition, but they cannot be considered fully similar anymore when reducing the element separation, hence differing more and more from the array response properties desired in the algorithms. (Fig. 2).

The antenna patterns were then measured in a near-field measurement system, for both 0.4 λ and 0.2 λ mutual distance (Fig. 3). Differences between simulation and measurement are mostly due to the simplified structure of the guiding rails used in simulation. Also in this case, bigger differences among the patterns of central and edge antennas are observed at smaller mutual distances.

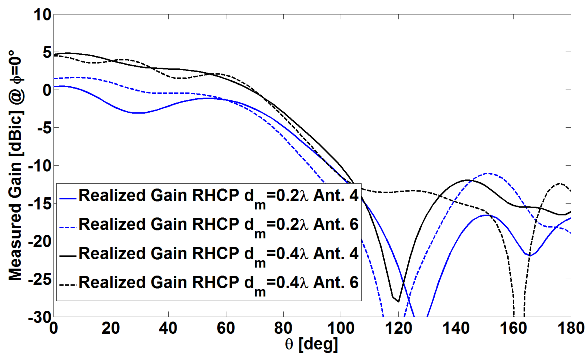


Figure 3. Antenna pattern of antennas no. 4 and 6 of the 6-element DRA array as measured in the Satimo Starlab anechoic chamber (f=1575 MHz, $\phi = 0°$cut)

IV. Array Interpolation Results

For the DOA estimations in the simulations $N = 100$ snapshots are used and the Root Mean Squared Error (RMSE) is calculated with respect to 1000 Monte Carlo simulations. Two signals from $\theta_1 = 45°$ and $\theta_2 = 15°$ with a mutual Pearson product-moment correlation coefficient of 1 are impinging on the array. The Signal-to-Noise-Ratio (SNR) is defined as $\mathrm{SNR} = \frac{\sigma_1^2}{\sigma_n^2} = \frac{\sigma_2^2}{\sigma_n^2}$. The given RMSE is

$$\mathrm{RMSE} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left((\hat{\phi}_{1,k} - \phi_1)^2 + (\hat{\phi}_{2,k} - \phi_2)^2\right)}, \quad (4)$$

where $\hat{\phi}_{i,k}$ is the estimate of ϕ_i at the k-th Monte Carlo run.

Fig. 4 presents the results of the mentioned interpolation methods for the measured array response with a 0.4 λ element separation. In this scenario the linear approach provides already very low DOA estimation bias, as the mutual coupling between antenna elements is not very strong. The nonlinear approach provides improved performance at the cost of a much higher computational complexity. Furthermore, if interpolation is not applied, a large DOA estimation bias is present at the final estimation even for high SNRs.

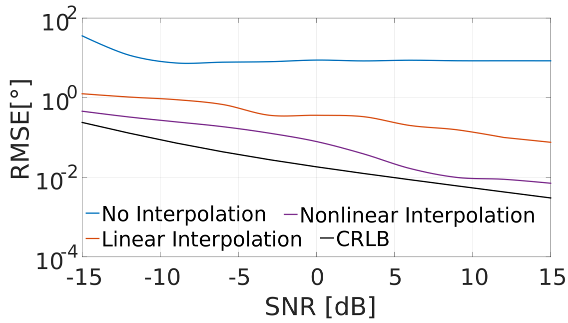


Figure 4. RMSE for the DOA results, obtained with measured data from the array having 0.4 λ element separation

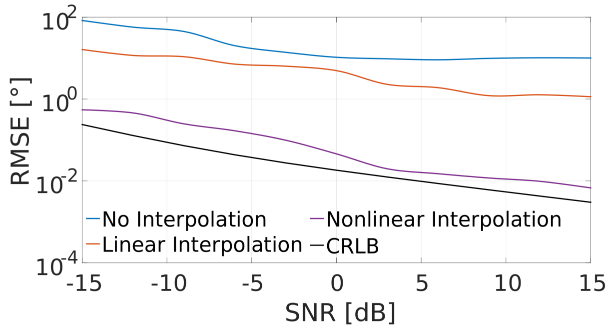


Figure 5. RMSE for the DOA results, obtained with simulated data from the array having 0.2 λ element separation

On the other hand, if a 0.2 λ element separation is taken into consideration, the results shown in Fig. 5 are obtained. A consistent increase in the RMSE also when performing linear interpolation shows how, in this case, the use of a nonlinear interpolation becomes mandatory if precise DOA is needed for low or negative SNRs. Results with measured data at 0.2 λ element separation will be shown at the conference.

V. Conclusions

In this work, the design and manufacturing of an array with adjustable mutual distance between the elements has been shown. The array has been used to evaluate the performance of the direction of arrival estimation, when using a linear and nonlinear array interpolation approach, without suffering from manufacturing repeatability issues. The results show that when antenna elements are separated by larger distances, resulting in less mutual coupling, the linear array interpolation approach is capable of providing a high degree of accuracy with low computational complexity. However, as the distance between antenna elements is made smaller, the performance of the linear approach is heavily impacted and other approaches, such as the nonlinear one, become necessary.

References

[1] G. X. Gao, M. Sgammini, M. Lu and N. Kubo, "Protecting GNSS Receivers From Jamming and Interference," *Proc. of the IEEE*, vol. 104, no. 6, pp. 1327-1338, June 2016.

[2] M. Cuntz, A. Konovaltsev and M. Meurer, "Concepts, Development, and Validation of Multiantenna GNSS Receivers for Resilient Navigation," *Proc. of the IEEE,* vol. 104, no. 6, pp. 1288-1301, June 2016.

[3] C. Guenther, "A survey of spoofing and counter-measures," *Navigation* , vol. 61, no. 3, pp. 159–177, 2014.

[4] N. Basta et al., "System Concept of a Compact Multi-Antenna GNSS Receiver", *Proc. of 2012 German Microwave Conf. (GEMIC 2012),* Ilmenau (Germany), Mar. 2012

[5] S. Caizzone, G. Buchner, W. Elmarissi, "Miniaturized Dielectric Resonator Antenna Array for GNSS Applications", *Int. Journ. Antennas Propag.*, vol. 2016, pp.1-10, Jul. 2016

[6] A. J. Barabell, "Improving the Resolution Performance of Eigenstructured Based Direction-Finding Algorithms," *Proc. ICASSP*, 1983.

[7] M. Haardt and J. A. Nossek, "Unitary ESPRIT: how to obtain increased estimation accuracy with a reduced computational burden," *IEEE Trans. Signal Proc.*, vol. 43, no. 5, May 1995.

[8] J. E. Evans, J. R. Johnson, and D. F. Sun, "Application of advanced signal processing techniques to angle of arrival estimation in ATC navigation and surveillance system," *MIT Tech. Rep.*, 1982.

[9] S. Pillai and B. H. Kwon, "Forward/backward Spatial Smoothing Techniques for Coherent Signal Identification ," *IEEE Trans. Acoustics, Speech and Signal Proc*., vol. 37, pp. 8–9, January 1989.

[10] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *J. Royal Stat. Soc. Series B*, vol. 39, no. 1, pp. 1-38, 1977.

[11] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans.Antennas Propag.*, vol. 34, no. 3, pp. 276-280, Mar 1986.

[12] M. A. M. Marinho, F. Antreich, J. P. C. L. da Costa, and J. A. Nossek, "A Signal Adaptive Array Interpolation Approach with Reduced Transformation Bias for DOA estimation of Highly Correlated Signals," *Proc. ICASSP,* 2014.

[13] M. A. M. Marinho et al., "Array interpolation based on multivariate adaptive regression splines," *Proc. SAM Workshop*, Rio de Janeiro (Brazil), 2016, pp. 1-5.

[14] A. Petosa, A. Ittipiboon, "Dielectric Resonator Antennas: a Historical Review and the Current State of the Art", *IEEE Antennas Propag. Mag.,* vol. 52, no. 5, pp.91-116, Oct. 2010

[15] Friedman, Jerome H. "Multivariate adaptive regression splines." *Annals of statistics* , pp. 1-67, 1991