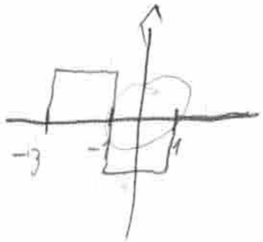
2.4-2

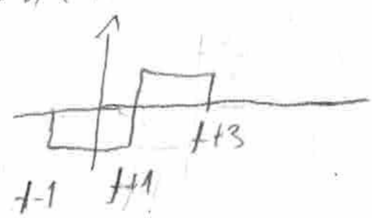
$t=0$

$t=1$

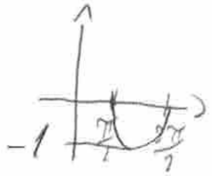
$h(t)$



$h(\tau)$



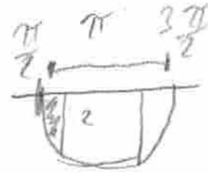
$x(t)$



$t-1 < \frac{3\pi}{2}$

$t = \frac{3\pi}{2} + 1$

$t = \frac{3\pi+2}{2}$



$\frac{\pi+1}{2}$

(b)

$t-1 = \frac{\pi}{2} + \frac{\pi-2}{2} = \pi - 1$

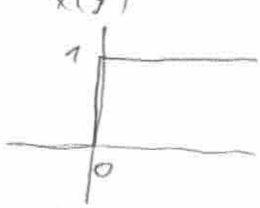
$t = \pi$

$h(t) = 1-(t-1)^2 \quad 1-t^2+2t+1$

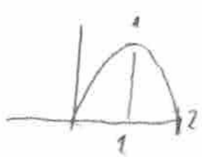
$h(t) = 2t - t^2$

2.4-5

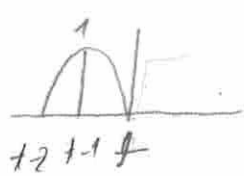
$x(t)$



$h(t)$



$h(t-\tau)$



(a) yes $h(t)$ an $x(t)$ are 0 for $t<0$

(b) $t<0 = 0$

$0<t<2 = \int_0^t 2\tau - \tau^2 = \frac{2\tau^2}{2} - \frac{\tau^3}{3}\Big|_0^t = t^2 - \frac{t^3}{3}$

8 ... 16 ~~16~~ 12-8 4

2.5-1

a) $\lambda^2 + 8\lambda + 12 \to \lambda = -6, -2$

Stable in both senses, roots on LHP

b) $\lambda^3 + 3\lambda^2 + 2\lambda \to \lambda = -2, -1, 0$

Marginally stable, root on imaginary axis; BIBO unstable

c) $\lambda^4 + 2\lambda^2 \to \lambda = [0, 0, \pm\sqrt{2}j]$

Unstable, repeated roots on imaginary axis; BIBO unstable

d) $(\lambda+1)(\lambda^2 - 6\lambda + 5) = \lambda^3 - \cancel{6}^{5}\lambda^2 + \cancel{5}\lambda + \lambda^2 - \cancel{6}^{1}\lambda + 5 = \lambda^3 - 5\lambda^2 - 1\lambda + 5$

$\lambda = [-1, 1, 5]$

Unstable, roots on RHP; BIBO unstable

2.5-2

a) $(\lambda+1)(\lambda^2 + 2\lambda + 5)^2 = (\lambda+1)(\lambda^2+2\lambda+5)(\lambda^2+2\lambda+5)$

$\lambda = [-1, -1 \pm 2i, -1 \pm 2i]$

Stable in both senses, roots on RHP

b) $(\lambda+1)(\lambda^2+9)$

$\lambda = [-1; \pm 3j]$

Marginally stable, roots on imaginary axis; BIBO unstable

c) $(\lambda+1)(\lambda^2+9)^2 = (\lambda+1)(\lambda^2+9)(\lambda^2+9)$

$\lambda = [-1, \pm 3j, \pm 3j]$

Unstable, repeated roots on imaginary axis; BIBO unstable

d) $(\lambda+1)(\lambda^2+4)(\lambda^2+9) \to \lambda = [\pm 1; \pm 2i; \pm 3i]$

$$\int_{-\infty}^{\infty} e^{t}\left[\frac{2}{3}\cos\left(\frac{2}{3}t\right)+\frac{1}{3}\sin(\pi t)\right]u(123-t)$$

$$\frac{2}{3}\int_{-\infty}^{123} e^{t}\cos\left(\frac{2}{3}t\right)+\frac{1}{3}\int_{-\infty}^{123} e^{t}\sin(\pi t)$$

$$\frac{2}{3}\left[\frac{e^{t}\left(\frac{2}{3}\sin\left(\frac{2}{3}t\right)+\cos\left(\frac{2}{3}t\right)\right)}{\left(\frac{2}{3}\right)^2+1}\right]_{-\infty}^{123}+\frac{1}{3}\left[\frac{e^{t}\left(\sin(\pi t)-\pi\cos(\pi t)\right)}{1+\pi^2}\right]_{-\infty}^{123}$$

$$\underbrace{\frac{2}{3}\left[\frac{e^{123}\left(\frac{2}{3}\sin\left(\frac{2}{3}123\right)+\cos\left(\frac{2}{3}123\right)\right)}{\left(\frac{2}{3}\right)^2+1}\right]+\frac{1}{3}\left(\frac{e^{123}\left(\sin(123\pi)-\pi\cos(123\pi)\right)}{1+\pi^2}\right)}_{g(t)}$$

$g(t)<\infty$, the system is BIBO stable

$$|e^{t}|\,\left|\frac{2}{3}\cos\left(\frac{2}{3}t\right)+\frac{1}{3}\sin(\pi t)\right|$$

$$-\frac{2}{3}\le f(t)\le\frac{2}{3}\qquad -\frac{1}{3}\le g(t)\le\frac{1}{3}$$

$$u(t)\le 1$$

$$\int_{-\infty}^{\infty}|e^{t}|\,u(123-t)\le\int_{-\infty}^{\infty}e^{t}\left[\frac{2}{3}\cos\left(\frac{2}{3}t\right)+\frac{1}{3}\sin(\pi t)\right]u\ldots$$

$$\int_{-\infty}^{123}e^{t}=e^{123}$$

2.5-3

$$\int_{-\infty}^{\infty} |e^{t}| \left| \frac{2}{3}\cos\left(\frac{3}{2}t\right) + \frac{1}{3}\sin(\pi t) \right| u(123-t)\, dt = g(t)$$

$\cos(x) \leq 1$, ≥ -1 $\qquad$ $\sin(x) \leq 1$, ≥ -1

$$g(t) \leq \int_{-\infty}^{\infty} |e^{t}| \, |\pm 1| \, u(123-t)\, dt$$

$$\int_{-\infty}^{123} e^{t} = e^{123} < \infty$$

$$g(t) < \infty$$

System is BIBO stable

2.5-4

a) for $T \geq 0$ $u(t-T)=0$ for $t<0$

b) $\int_{-\infty}^{\infty} |\frac{1}{t}| u(t-T)\, dt = \int_{T}^{\infty} |\frac{1}{t}|\, dt = \ln|t| \Big|_{T}^{\infty} = \ln\infty - \ln|T|$

there is no T for which $\ln\infty - \ln|T| < \infty$

2.5-6

a) $h(t) = u(t) = e^{0} u(t)$ $\lambda = [0]$

b) Marginally stable

d) BIBO unstable

e) $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) u(t-\tau) d\tau = \int_{-\infty}^{t} x(\tau) d\tau$

The system can be used as an integrator