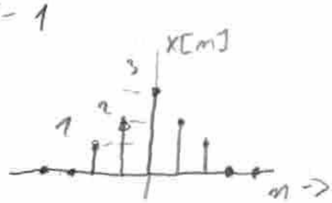
3.1-1

a)

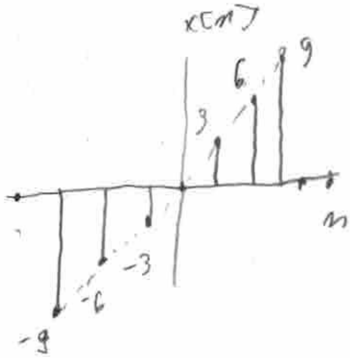


$$\sum_{n=-\infty}^{\infty} |x[n]|^2 \to x[n] < 0 \ \forall -2 > n > 2$$

$$\sum_{n=-2}^{2} |x[n]|^2 = 1+2^2+3^2+2^2+1 = 19.$$

b) Signal is the same as a but shifted; energy does not change = 19

c)



$$\sum_{n=-3}^{3} |x[n]|^2 = 2(9^2+6^2+3^2) = 252$$

Signal is odd

d) $2(4^2+2^2) = 40$

3.1-2)

a) Period = 6   Power = $\frac{1}{6}\sum_{n=-3}^{3} |x[n]|^2 = \frac{1}{6}(1^2+2^2+3^2+2^2+1^2) = \frac{19}{6}$

b) Period = 12   Power = $\frac{1}{12}\sum_{n=-6}^{6} |x[n]|^2 = \frac{1}{12}(3^2+2^2+1^2+1^2+2^2+3^2) = \frac{28}{12} = \frac{7}{3}$

c) $\frac{1}{N_0}\sum_{n=0}^{N_0-1} a^n = \frac{a^{N_0}-1}{N_0(a-1)}$

3.1-3

$$\frac{1}{N_0}\sum_{n=0}^{N_0-1}\left|D e^{j(2\pi/N_0)^n}\right|^2 = \frac{1}{N_0}\sum_{n=0}^{N_0-1}\left(|D|\,|e^{j(\frac{2\pi}{N_0})^n}|\right)^2 =$$

$$= \frac{1}{N_0}\sum_{n=0}^{N_0-1}\left(|D|\sqrt{\cos^2\left(\left(\tfrac{2\pi}{N_0}\right)^n\right)+\sin^2\left(\left(\tfrac{2\pi}{N_0}\right)^n\right)}\right)^2 = \frac{1}{N_0}\sum_{n=0}^{N_0-1}(|D|\cdot 1)^2 =$$

$$\frac{N_0|D|^2}{N_0} = |D|^2$$

$$\frac{1}{N_0}\sum_{n=0}^{N_0-1}\left|\sum_{r=0}^{N_0-1} D_r e^{jr\overbrace{(\frac{2\pi}{N_0})}^{w_0}n}\right|^2 = \frac{1}{N_0}\sum_{n=0}^{N_0-1}\left(\sum_{r=0}^{N_0-1} D_r e^{jrw_0 n}\sum_{m=0}^{N_0-1} D_m^* e^{-jmw_0 n}\right)$$

$$= \frac{1}{N_0}\sum_{r=0}^{N_0-1}\sum_{m=0}^{N_0-1} D_r D_m^* \sum_{n=0}^{N_0-1} e^{jw_0 n(r-m)} = \frac{1}{N_0}\sum_{r=0}^{N_0-1}\sum_{m=0}^{N_0-1} D_r D_m^* \cdot N_0$$

$$= \sum_{r=0}^{N_0-1}|D_r|^2$$

3.1-4

a) $x[n] = x[-n]$ for even signal ; therefore $x[n] = \frac{1}{2}[x[n]+x[-n]]$

$x[n] = -x[-n]$ for odd signal ; therefore $x[n] = \frac{1}{2}[x[n]-x[-n]]$

$x_e[n] = \frac{1}{2}[0{,}8^n u[n] + 0{,}8^{-n} u[-n]]$ $\quad x_o[n] = \frac{1}{2}[0{,}8^n u[n] - 0{,}8^{-n} u[-n]]$

b) $$E_{x_e} = x[0] + \sum_{n=-\infty}^{-1}\left|\tfrac{1}{2}0{,}8^{-n}u[-n]\right|^2 + \sum_{n=1}^{\infty}|0{,}8^n u[n]| = 1+\frac{1}{4}\sum_{n=-\infty}^{-1}0{,}64^{-n}+\frac{1}{4}\sum_{n=1}^{\infty}0{,}64^n$$

$$= 1+\frac{1}{2}\sum_{n=1}^{\infty}0{,}64^n = 1+\frac{1}{2}\cdot\left[\frac{0{,}64}{1-0{,}64}\right] = 1{,}89$$

$$E_{x_{odd}} = \sum^{\infty}\left|\tfrac{1}{2}0{,}8^n\right|^2 + \sum^{-1}\left|\tfrac{1}{2}-0{,}8^n\right|^2 = \frac{1}{4}\sum_{n=1}^{\infty}|0{,}8^n|^2 + \frac{1}{4}\sum_{n=-\infty}^{-1}|-0{,}8^n|^2$$

c)

$$E_{x\,even} = |x[0]|^2 + \frac{1}{4}\sum_{n=1}^{\infty}|x[n]|^2 + \frac{1}{4}\sum_{n=-1}^{-\infty}\underbrace{|x[-n]|^2}_{\hookrightarrow = x[n]} = |x[0]|^2 + \frac{1}{2}\sum_{n=1}^{\infty}|x[n]|^2$$

$$E_{x\,odd} = \frac{1}{4}\sum_{n=1}^{\infty}|x[n]| + \frac{1}{4}\sum_{n=-1}^{-\infty}\underbrace{|-x[-n]|^2}_{\hookrightarrow = x[n]} = \frac{1}{2}\sum_{n=1}^{\infty}|x[n]|^2$$

$$E_{x\,even} + E_{x\,odd} = \sum_{n=0}^{\infty}|x[n]|^2 = E_x$$

3.1-5

Omiting $u[n]$ for simplicity

a) $x_e[n] = \frac{1}{2}[x[n] + x[-n]]$ $\quad x_o[n] = \frac{1}{2}[x[n] - x[-n]]$

$$E_{x_e} = x[0] + \frac{1}{4}\sum_{n=1}^{\infty}[x[n]|^2 + \frac{1}{4}\sum_{n=1}^{\infty}\underbrace{|x[-n]|^2}_{\hookrightarrow = x[n]} = E_{x_e} = |x[0]|^2 + \frac{1}{2}\sum_{n=1}^{\infty}|x[n]|^2$$

$$E_{x_o} = \frac{1}{4}\sum_{n=1}^{\infty}|x[n]|^2 + \frac{1}{4}\sum_{n=1}^{\infty}\underbrace{|-x[-n]|^2}_{\hookrightarrow |-x[-n]| = |x[n]|} = \frac{1}{2}\sum_{n=1}^{\infty}|x[n]|^2$$

$$E_{x_e} + E_{x_o} = |x[0]|^2 + \sum_{n=1}^{\infty}|x[n]|^2 = \sum_{n=0}^{\infty}|x[n]|^2$$

b)

$$E_{x_e, x_o} = \sum_{n=-\infty}^{\infty} x_e[n]\,x_o^*[n] = \frac{1}{4}\sum_{n=0}^{\infty}(x[n] + x[-n])(x[n] - x[-n])^*$$

$$\frac{1}{4}\sum_{n=0}^{\infty}(|x[n]|^2 - |x[0]|^2 + |x[0]|^2 - |x[-n]|^2)$$

$$= \frac{1}{4}\sum_{n=0}^{\infty}|x[n]|^2 - \frac{1}{4}\sum_{n=0}^{\infty}|x[-n]|^2 = 0$$

3;1-6

$$x[n] \begin{cases} \left(\frac{1}{3}\right)^n & n \geq 0 \\ A^n & n < 0 \end{cases}$$

$$E_x = \sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n\right|^2 + \sum_{n-\infty}^{-1} |A^n|^2 =$$

$$\longrightarrow \sum_{n=-1}^{\infty} (A^2)^n = \frac{(A^{-2})^{\infty} - 1}{1 - A^2}$$

$$\hookrightarrow \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{1}{1-\frac{1}{9}}$$

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \left(\frac{(A^{-2})^n - 1}{1 - A^2} + \frac{1}{1 - 1/9} \right)$$

a) $A = \frac{1}{2}$ $\quad \frac{(A^{-2})^{\infty} - 1}{1 - A^2} = \frac{2^{\infty} - 1}{1 - \frac{1}{4}} = \infty$ $\quad$ not energy nor power

b) $A = 1$ $\quad \sum_{n=-1}^{\infty} (A^2)^n = \sum_{n=-1}^{\infty} 1 = \infty$ $\quad$ Power signal

$$\lim_{N \to \infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 1 + \frac{1}{1-1/9} \right) = \lim_{n \to \infty} \frac{N}{2N+1} + \frac{\frac{1}{1-1/9}}{2N+1} = \lim_{N \to \infty} \frac{1}{2+\frac{1}{N}} = \frac{1}{2}$$

c) $A = 2$

$$\sum_{n=-1}^{-\infty} (A^2)^n = \frac{(A^{-2})^{\infty} - 1}{1 - A^2} = \frac{\left(\frac{1}{4}\right)^{\infty} - 1}{1 - 4} = \frac{1}{3} \qquad E_x = \frac{1}{3} + \frac{1}{1-1/9} = \frac{35}{24}$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \left(\frac{35}{24} \right) = 0$$

Energy Signal

3.1-7

$$x[n] = \mathrm{Re}\{3(e^{j(\frac{\pi}{4})})^n\} = \mathrm{Re}\{3e^{j(\frac{\pi}{4})n}\} = 3\cos(\frac{\pi}{4}n)$$

$T \Rightarrow \frac{\pi}{4} \cdot n = 2\pi \quad n = 8$

$$P_x = \frac{1}{8}\sum_{n=0}^{7}|3\cos(\frac{\pi}{4}n)|^2 = \frac{1}{8}[3^2 + (\frac{\sqrt{2}}{2}\cdot 3)^2 + 0 + (\frac{\sqrt{2}}{2}\cdot 3)^2 + 3^2 + (\frac{\sqrt{2}}{2}\cdot 3)^2 + 0 + (\frac{\sqrt{2}}{2}\cdot 3)^2] = \frac{1}{8}[18+18] = \frac{9}{2}$$

3.2-2

a) $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=N}^{N} |-x[n]|^2 \rightarrow |-x[n]| = |x[n]|$

$= \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=N}^{N} |x[n]|^2 = P_x$

b) $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[-n]|^2$; let $m=-n$ $= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} |x[m]|^2 = P_x$

c) $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n-m]|^2$; let $n-m=y$ $= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{y=-N}^{N} |x[y]|^2 = P_x$

d) $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |c\,x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} |c|^2 \sum_{n=-N}^{N} |x[n]|^2 = |c|^2 P_x$

e) P_x, following same as c.

3.2-4

$x[n] = [\overset{-1}{-1}, \overset{0}{2}, -3, 4, -5, 4, -3, \overset{7}{2}, -1]$

a) $E_x = \sum_{n=-1}^{7} |x[n]|^2 = 85$; $P_x = 0$

b) $y[n] = [2, 4, \overset{\downarrow}{4}, 2]$

$y[0] = x[4]$

$y[-1] = x[2]$

$y[-1] = x[0]$

$y[-3] = x[-2]$

c) $[-1, 4, 0, 0, -5, 0, 0, \overset{10}{4}, 0, 0, -3, 0, 0, 2, 0, 0, -1]$

$z[0] = x[2]$

3.2-6)

a) $E_x = \sum_{n=0}^{5} |x[n]|^2 = 91, \quad P_x = 0$

b)

$$\frac{N_1 \cdot 0}{N_2} + N_3 = 4 \Rightarrow N_3 = 4$$

$N_2 = 3$ -> Interpolated signal with 3x upsampling

$$y[3] = x[2] \Rightarrow \frac{3N_1}{3} + 4 = 2 \Rightarrow N_1 = -2$$

c) $\tilde{y}[0] = y[-6] + y[0] + y[6] + y[12] = 5+1$

$\tilde{y}[1] = y[1] + y[7] = 0$

$\tilde{y}[2] = y[2] + y[8] = 0$

$\tilde{y}[3] = y[3] + y[9] = 3$

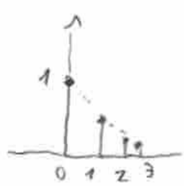
$\tilde{y}[4] = y[4] + y[10] = 0$

$\tilde{y}[5] = y[5] + y[11] = 0$

$$P_y = \frac{1}{6}\sum_{n=0}^{5} |\tilde{y}|^2 = \frac{1}{6}\cdot(36+9) = \frac{45}{6} = \frac{15}{2}$$

3.2-10)

$x[n] =$



a) $y_a[0] = 1; \quad y_a[1] = \left(\frac{1}{2}\right)^2$

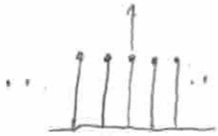
b) $y_b[0] = 1, \quad y_b[3] = \frac{1}{2}$

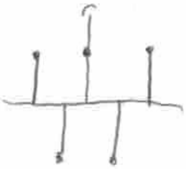
c) $y_c[0] = \frac{1}{2}; \quad y_c[1] = \left(\frac{1}{2}\right)^4$

d) $y_d[2] = \frac{1}{2}; \quad y_d[1] = \left(\frac{1}{2}\right)^3$

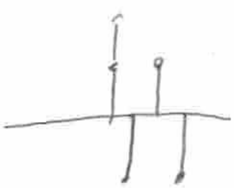
e) $y_e[-8] = 1; \quad y_e[-10] = \frac{1}{2}$

3.3 - 1)

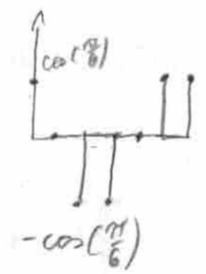
a)  $P_x = 1$

b)  $P_x = 1$

c)  $P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \lim_{N\to\infty} \frac{N+1}{2N+1} = \frac{1}{2}$

d)  $P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |(-1)^n u[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \frac{1}{2}$

e) $\cos\left(\frac{\pi}{3} n + \frac{\pi}{6}\right)$; $T = 6$



$P_x = \frac{1}{6} \sum_{n=0}^{5} \cos\left(\frac{\pi}{3} n + \frac{\pi}{6}\right)$

$= \frac{1}{6}\left[\left(\frac{\sqrt{3}}{2}\right)^2 + 0 + \left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 + 0 + \left(\frac{\sqrt{3}}{2}\right)^2\right]$

$= \frac{1}{2}$

3.3 - 2)

a) $u[n] - u[n-2] = \begin{cases} 1; & n = 0,1 \\ 0; & n \neq 0,1 \end{cases}$

$= \delta[n] + \delta[n-1]$

b) $2^{n-1} \sin\left(\frac{\pi n}{3}\right) u[n]$

$\frac{2^n}{2} = 2^{n-1}$ $\quad \sin\left(\frac{\pi n}{3}\right) = 0$ for $n = 0$

c) $n(n-1) = 0$ for $n = 0, 1$

$n(n-1)\gamma^n u[n] = n(n-1)\gamma^n u[n-2]$

d) for n odd $u[n] + (-1)^n u[n] = 0$

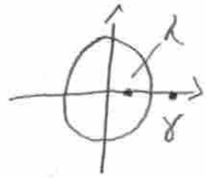
for n even $\sin\left(\frac{\pi n}{2}\right) = 0$

e) for n even $u[n] + (-1)^{n+1} u[n] = 0$

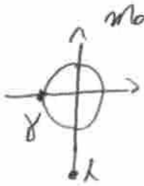
3.3-7)

a) $0{,}6067^n$ stable

b) $1{,}649^n$ unstable



c) -1^n marginaly stable



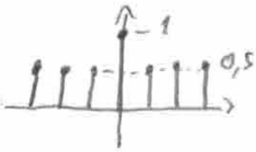
d) -1^n marginally stable

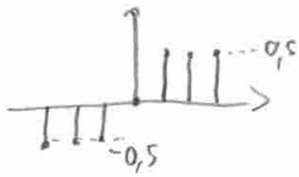
3.3-9)

$$x[n] = \frac{1}{2}\left[x[n] + x[-n]\right] + \frac{1}{2}\left[x[n] - x[-n]\right]$$

a)

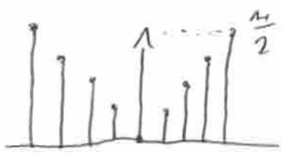
$$u[n] = \frac{1}{2}\left[u[n] + u[-n]\right] + \frac{1}{2}\left[u[n] - u[-n]\right]$$

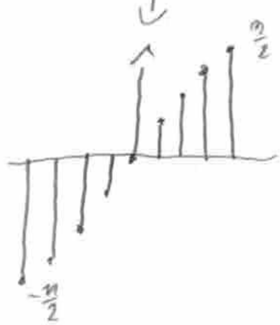




b)

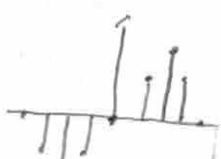
$$n\,u[n] = \frac{n}{2}\left[u[n] + u[-n]\right] + \frac{n}{2}\left[u[n] - u[-n]\right]$$





c)

$$\sin\left(\frac{n\pi}{4}\right) = \frac{1}{2}\left[\sin\left(\frac{n\pi}{4}\right) + \sin\left(-\frac{n\pi}{4}\right)\right] + \frac{1}{2}\left(\sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right)\right)$$

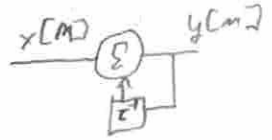


d) $\cos(\ldots)$ $\frac{1}{2}\left[\cos\left(\frac{n\pi}{4}\right) + \cos\left(-\frac{n\pi}{4}\right)\right] + \frac{1}{2}\left[\cos\left(\frac{\pi n}{4}\right) - \cos\left(-\frac{\pi n}{4}\right)\right]$

3.4-1)

a) $y[n] = x[n] + x[n-1] + \dots$

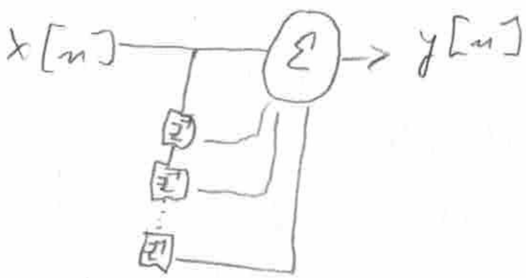
b) $y[n] = y[n-1] + x[n]$



3.4-3)

a) $y[n] = \frac{1}{5}\left(x[n] + x[n-1] \dots + x[n-4]\right)$

b)



3.4-4) $y[n] = T x[n] + y[n-1]$

$y[n] = T u[n] + \underline{y[n-1]}$

$\hookrightarrow T u[n] + y[n-2]$

$y[n] = T \sum_{m=0}^{n} u[n] \Rightarrow y[n] = T(n+1)u[n]$

3.4-5)

$y''[n] + a_1 y'[n] + a_0 y[n] = x[n]$

$y'[n] = \frac{y[n] - y[n-1]}{T}$

$y''[n] = \frac{\frac{y[n]-y[n-1]}{T} - \frac{y[n-1]-y[n-2]}{T}}{T}$

3.4-7)

a)

b) $\sum_{n=-\infty}^{\infty} |h[n]| = \infty$; not stable

c) $h[n] \neq 0 \ \forall n \neq 0$

not memoryless

d) $h[n] = 0 \ \forall n < 0$

causal

3.4-9)

a) True

b) False let $x[n] = n^2$

$\sum_{n=0}^{\infty} |n^2|^2 = \infty \quad \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} |n^2|^2 = \infty$

c) True

d) False if $m = n-1$ $\quad y[m] = x[m+1]$

output depends on future inputs

e) False

let $x[n] = \delta[n-2] \ \& \ E_x = 1$

let $a = 3 \rightarrow E_x = 0$

3.4-11) $y[n] = \frac{1}{2} \sum_{k=-\infty}^{\infty} x[k] (\delta[n-k] + \delta[n+k])$

a)

$y[n] = \frac{1}{2} (x[n] + x[-n]) \rightarrow x$ even

b) for $x[n] \leq M_x < \infty \quad y[n] \leq \frac{1}{2}(M_x + M_x) = M_x < \infty$

$x[-n] \leq M_x < \infty$

BIBO stable

c) Aditivity $\quad y_1[n] = \frac{1}{2} \sum_{k=-\infty}^{\infty} (x_1[k] + x_2[k]) (\delta[n-k] + \delta[n+k])$

$y[n] = \frac{1}{2} (x_1[n] + x_1[-n] + x_2[n] + x_2[-n]) = y_1[n] + y_2[n]$

Homogeneity $\quad y[n] = \frac{1}{2} \sum_{k=-\infty}^{\infty} a x[n] (\delta[n+k] + \delta[n-k])$

d) No; since at time instant n the output depends on $-n$

e) No; for $n > 0$ the system relies on $-n$ which is < 0

f)

$$\sum_{k=-\infty}^{\infty} x[k] \left(\delta[n-N_0-k] + \delta[n-N_0+k] \right)$$

$$x[n-N_0] + x[-(n-N_0)]$$ → This is only true for general even signals when $N_0 = 0$. Thus the system is not time invariant

10-3

3.4-16)

a) $T = \frac{1}{60}$ $\quad v[n] = k(x[n] - x[n-1])$ $\quad K = 60s^{-1}$

b)

$$v[n] = \frac{x[n] - x[n-1]}{T} = k(x[n] - x[n-1])$$

$$a[n] = k\left(k(x[n] - x[n-1]) - k(x[n-1] - x[n-2])\right)$$

$$= k^2(x[n] - 2x[n-1] - x[n-2])$$

$$h[n] = k^2(\delta[n] - 2\delta[n-1] - \delta[n-2])$$