3.5-1)

$2y[n] + 2y[n-1] = x[n-1]$

a) $2(E+1)\,y[n] = E\,x[n]$

b) $2h[0] + 2h[-1] = \delta[-1]$

$h[0] = \frac{\delta[-1]}{2} - h[-1] = 0$

$h[1] = \frac{1}{2} - 0 = \frac{1}{2}$

$h[2] = 0 - \frac{1}{2} = -\frac{1}{2}$

$h[3] = 0 - (-\frac{1}{2}) = \frac{1}{2}$

$h[4] = 0 - \frac{1}{2} = -\frac{1}{2}$

c) $y[0] = 0 - 0 = 0 \rightarrow y[n] = \frac{1}{2}x[n] - y[n-1]$

↳ 0 for zero state

$y[1] = 1 - 0 = 1$

$y[2] = 1 - 1 = 0$

$y[3] = 1 - 0 = 1$

$y[4] = 1 - 1 = 0$

d) $y[0] = \frac{1}{2}x[-1] - y[-1] = 0 - 1 = -1$

$y[1] = 0 - [-1] = 1$

$y[2] = 0 - 1 = -1$

$y[3] = 0 - (-1) = 1$

$y[4] = 0 - 1 = -1$

3.5-2)

a) $y[n+1] - \frac{1}{2}y[n] = 0$ $\qquad y[-1] = 10$

$y[n] - \frac{1}{2}y[n-1] = 0$

$y[n] = \frac{1}{2}y[n-1]$

$\to y[0] = \frac{1}{2}(10) = 5$

$y[1] = \frac{1}{2}(5) = \frac{5}{2}$

$y[2] = \frac{1}{2}\left(\frac{5}{2}\right) = \frac{5}{4}$

b) $y[n+1] + 2y[n] = x[n+1]$ $\qquad x[n] = e^{-n}u[n]$

$y[-1] = 0$

$y[n] = x[n] - 2y[n-1]$

$y[0] = 1 - 2(0) = 1$

$y[1] = e^{-1} - 2(1) = e^{-1} - 2$

$y[2] = e^{-2} - 2(e^{-1} - 2) = e^{-1}(e^{-1} - 2) + 4$

3.5-3) $y[n] = 0{,}6y[n-1] + 0{,}16y[n-2]$; $y[-1] = -25$, $y[-2] = 0$

$y[0] = 0{,}6(-25) = -15$

$y[1] = 0{,}6(-15) + 0{,}16(-25) = -13$

$y[2] = 0{,}6(-13) + 0{,}16(-15) = -10{,}2$

3.5-5) $y[n] = x[n] + 3x[n-1] + 3x[n-2] - 3y[n-1] + 2y[n-2]$

$y[0] = 3^0 + 3\cdot 0 + 3\cdot 0 - 3(3) - 2(2) = -12$

$y[1] = 3^1 + 3(1) + 3\cdot(0) - 3(-12) - 2(3) = 36$

$y[2] = 3^2 + 3(3) + 3(1) - 3(36) - 2(-12) = -63$

3.6-1 $E^2 + \frac{E}{6} - \frac{1}{6} = 0$ $\lambda = -\frac{1}{2}, \frac{1}{3}$

$$c_1\left(-\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$$

$$c_1\left(-\frac{1}{2}\right)^{-1} + c_2\left(\frac{1}{3}\right)^{-1} = 3$$

$$c_1\left(-\frac{1}{2}\right)^{-2} + c_2\left(\frac{1}{3}\right)^{-2} = -1$$

$$\begin{bmatrix} (-\frac{1}{2})^{-1} & (\frac{1}{3})^{-1} \\ (-\frac{1}{2})^{-2} & (\frac{1}{3})^{-2} \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{1}{3} \end{bmatrix}$$

$$-\left(-\frac{1}{2}\right)^n + \frac{1}{3}\left(\frac{1}{3}\right)^n$$

3.6-3 $E^2 + 2E + 1 = 0$ $\lambda = -1, -1$ $y[-1] = 1$ $y[-2] = 1$

$$c_1(-1)^n + c_2 n(-1)^n$$

$$c_1\left(\frac{1}{-1}\right) + c_2(-1)\left(\frac{1}{-1}\right) = -c_1 + c_2 = 1 \Rightarrow c_2 = c_1 + 1$$

$$c_1\left(\frac{1}{-1^2}\right) + c_2(-2)\left(\frac{1}{(-1)^2}\right) = c_1 - 2c_2 = 1$$

$$c_1 - 2(c_1 + 1) = 1$$

$$c_1 - 2c_1 - 2 = 1$$

$$-c_1 = 3$$

$$c_1 = -3$$

$$c_2 = -3 + 1$$

$$c_2 = -2$$

$$(-3 - 2n)(-1)^n$$

3.6-5

Equation is of order N; longest delay $= n - N$

lowest of delay is $0 \rightarrow y[n]$

thus the equation is $E^N = 0$

N repeated roots 0

$$c_1(0)^n + c_2 n(0)^n + c_3 n^2 (0)^n \dots = 0$$

3.6-6

a) $f[n] = f[n-1] + f[n-2]$

b) $E^2 - E - 1 = 0 \qquad \lambda = [0{,}618,\ 1{,}61]$

The system is not stable, root on RHP $(1{,}61)$

c) $f[1] = 0 \quad f[2] = 1$

$$C_1(-0{,}618)^1 + C_2(1{,}61)^1 = 0 \qquad \Rightarrow \qquad C_1 = 0{,}726$$

$$C_1(-0{,}618)^2 + C_2(1{,}61)^2 = 1 \qquad\qquad C_2 = 0{,}278$$

$$f[n] = 0{,}726(-0{,}618)^n + 0{,}278(1{,}61)^n$$

$$f[50] = 0{,}726(-0{,}618)^{50} + 0{,}278(1{,}61)^{50}$$

$$f[1000] = 0{,}726(-0{,}618)^{1000} + 0{,}278(1{,}61)^{1000}$$