3.7-1

a) $y[n+1] + 2y[n] = x[n]$

$(E+2)\,y[n] = x[n]$

$\lambda = [-2]$

$h[n] = \frac{1}{2}\delta[n] + C_1(-2)^n$

$n = -1$ $\quad h[0] + 2h[-1] = \delta[-1]$ $\qquad \frac{1}{2} + C_1 = 0$

$h[0] = 0$ $\qquad C_1 = -\frac{1}{2}$

$h[n] = \frac{1}{2}\delta[n] + 1u[n]$

b) $y[n] + 2y[n-1] = x[n]$

$y[n+1] + 2y[n] = x[n+1]$

$(E+2)\,y[n] = x[n]$

$\lambda = [-2]$

$h[n] = C_1(-2)^n$

$h[0] + 2h[-1] = \delta[0]$

$h[0] = 1$

$C_1(-2)^0 = 1$

$C_1 = 1$

$h[n] = (-2)^n u[n]$ //

3.7-2

a) $(E^2+1)\,y[n] = (E + \frac{1}{2})\,x[n]$

$\lambda[-i, i]$

$h[n] = \frac{1}{2}\delta[n] + [C_1(i)^n + C_2(-i)^n]\,u[n]$

$C_1 i - (-\frac{1}{4} + \frac{i}{2})\,i = 1$

$C_1 i + \frac{i}{4} + \frac{1}{2} = 1$

$C_1 i = \frac{1}{2} - \frac{i}{4} \rightarrow C_1 = \frac{1}{2i} - \frac{1}{4}$

$C_1 = -\frac{1}{4} - \frac{i}{2}$ //

$h[0] + h[-2] = \delta[-1] + \frac{1}{2}\delta[-2]$

$h[0] = 0$

$h[1] + h[-1] = \delta[0] + \frac{1}{2}[-1]$

$h[1] = 1$

$\frac{1}{2} + C_1 + C_2 = 0 \rightarrow C_1 = -\frac{1}{2} - C_2$

$C_1 i - C_2 i = 1$

$(-\frac{1}{2} - C_2)\,i - C_2 i = 1$

$\frac{1}{i} = -\frac{1}{2} - 2C_2 \Rightarrow -2C_2 = \frac{1}{2} - i$

$C_2 = -\frac{1}{4} + \frac{i}{2}$ //

c) $(E^2 - \frac{1}{6}E - \frac{1}{6})\, y[n] = \frac{1}{3} x[n]$

$$\lambda = [-\tfrac{1}{3}, \tfrac{1}{2}]$$

$$h[n] = \frac{\frac{1}{3}}{-\frac{1}{6}} \delta[n] + [c_1(-\tfrac{1}{3})^n + c_2(\tfrac{1}{2})^n] u[n]$$

$$h[n] = -2\,\delta[n] + [c_1(-\tfrac{1}{3})^n + c_2(\tfrac{1}{2})^n] u[n]$$

$$h[0] - \tfrac{1}{6} h[-1] - \tfrac{1}{6} h[-2] = \tfrac{1}{3}\delta[-2] = 0 \to h[0] = 0$$

$$h[1] - \tfrac{1}{6} h[0] - \tfrac{1}{6} h[-1] = \tfrac{1}{3}\delta[-1] = 0 \to h[1] = 0$$

$$-2 + c_1 + c_2 = 0 \qquad -\frac{c_1}{3} + \frac{c_2}{2} = 0 \qquad \begin{bmatrix} 1 & 1 \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6/5 \\ 4/5 \end{bmatrix}$$

$$h[n] = -2\,\delta[n] + \left(\tfrac{6}{5}(-\tfrac{1}{3})^n + \tfrac{4}{5}(\tfrac{1}{2})^n\right) u[n]$$

d) $(E^2 + \frac{1}{4})\, y[n] = E^2 x[n]$

$$h[0] + \tfrac{1}{4} h[-2] = \delta[0] \to h[0] = 1$$

$$h[1] + \tfrac{1}{4} h[-1] = \delta[1] \Rightarrow h[1] = 0$$

$$\lambda = [-\tfrac{i}{2}; \tfrac{i}{2}]$$

$$h[n] = [c_1(-\tfrac{i}{2})^n + c_2(\tfrac{i}{2})^n] u[n]$$

$$c_1 + c_2 = 1 \qquad c_1 = \tfrac{1}{2}$$

$$-\tfrac{i}{2}c_1 + \tfrac{i}{2}c_2 = 0 \qquad c_2 = \tfrac{1}{2}$$

$$h[n] = [\tfrac{1}{2}(-\tfrac{i}{2})^n + \tfrac{1}{2}(\tfrac{i}{2})^n] u[n]$$

g) $\left(E^3+\frac{1}{2}E^2-\frac{1}{4}E-\frac{1}{8}\right)y[n]=E^3x[n]$

$h[0]=1$

$h[1]=-\frac{1}{2}$

$\lambda=[-\frac{1}{2},-\frac{1}{2},\frac{1}{2}]$

$h[2]=-\frac{1}{2}(h[1])+\frac{1}{4}(h[0])$

$h[n]=[(c_1+n c_2)(-\frac{1}{2})^n+c_3(\frac{1}{2})^n]$

$h[2]=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

$c_1+c_3=1$

$-\frac{1}{2}c_1-\frac{1}{2}c_2+\frac{1}{2}c_3=-\frac{1}{2}$

$\frac{1}{4}c_1+\frac{2}{4}c_2+\frac{1}{4}c_3=\frac{1}{2}$

$c_1=\frac{3}{4}$

$c_2=\frac{1}{2}$

$c_3=\frac{1}{4}$

$h[n]=\left[\left(\frac{3}{4}+n\frac{1}{2}\right)\left(-\frac{1}{2}\right)^n+\frac{1}{4}\left(\frac{1}{2}\right)^n\right]u[n]$

3.7-3) $(4E^2+1)y[n]=(8E^2+8E)x[n]$

a) Orden 2

b) $\lambda=[-\frac{i}{2};\frac{i}{2}]$

$4h[0]=8 \rightarrow h[0]=2$

$4h[1]+h[-1]=8\delta[1]+8\delta[0]$

$h[1]=2$

c) $h[n]=[c_1(-\frac{i}{2})^n+c_2(\frac{i}{2})^n]u[n]$

$c_1+c_2=2$

$-\frac{ic_1}{2}+\frac{ic_2}{2}=2$

$c_1=1+2i$

$c_2=1-2i$

$h[n]=[(1+2i)(-\frac{i}{2})^n+(1-2i)(\frac{i}{2})^n]$

3.7-5) $(E^2 - 6E + 9)\,y[n] = E\,x[n]$

a) Second order

b) $\lambda = [3, 3]$ $\quad (3)^n;\ n(3)^n$

c) $h[0] = \delta[-1] = 0$ $\qquad h[n] = [(c_1 + n c_2)3^n]\,u[n]$

$c_1 = 0$

$h[1] = \delta[0] = 1$

$3c_2 = 1$

$c_2 = \frac{1}{3}$

$$h[n] = \left[\frac{n}{3}(3)^n\right]u[n]$$

3.7-7) $y[n] = (E^m b_0 + E^{m-1} b_1 + \dots + b_N)\,x[n]$

a)

$h[n] = \delta[n]\,b_0 + \delta[n-1]\,b_1 + \dots + \delta[n-N]\,b_N$

b) $h[n] = 3\,\delta[n] - 5\,\delta[n-1] - 2\,\delta[n-3]$