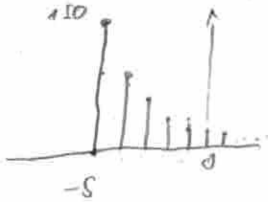
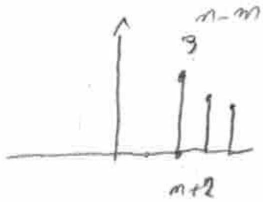
3.8-1)





$n+2 < -5$

$n < 7$

$$\sum_{m=-5}^{\infty} \frac{5}{(2)^m} \cdot (3)^{(n-m)} = 5 \cdot 3^n \sum_{m=-5}^{\infty} \frac{1}{2^m 3^m} \to \sum_{m=-5}^{\infty} \left(\frac{1}{6}\right)^m = \frac{\left(\frac{1}{6}\right)^{-5}\left(1-\left(\frac{1}{6}\right)^{\infty}\right)}{1-\frac{1}{6}}$$

$$= \frac{\left(\frac{1}{6}\right)^{-5}}{\frac{5}{6}} = \frac{6^6}{5}$$

$$5 \cdot 3^n \cdot \frac{6^6}{5} = 6^6 3^n$$

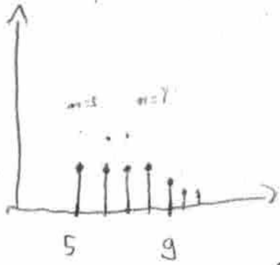
$n \geq 7$

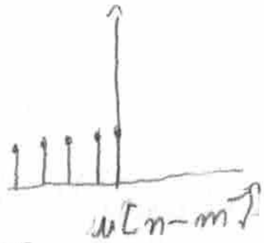
$$\sum_{m=n+2}^{\infty} \frac{5}{2^m} \cdot 3^{n-m} = 5(3)^n \sum_{m=n+2}^{\infty} \left(\frac{1}{6}\right)^m = 5(3)^n \cdot \left(\frac{1}{6}\right)^{n+2} \frac{\left(1-\left(\frac{1}{6}\right)^{\infty}\right)}{1-\frac{1}{6}} = \frac{5\, 3^n \left(\frac{1}{6}\right)^{n+2}}{\frac{5}{6}} =$$

$$= 3^n \cdot \left(\frac{1}{6}\right)^{n+1} = \cancel{3^n}\cancel{\left(\frac{1}{3}\right)^n}\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{n+1} = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^n = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right)^n$$

3.8-2)

a)





$n-0 < 5,$

$n < 5 \to 0$

$9 > n \geq 5$

$$\sum_{m=5}^{n} (u[m-5]-u[m-9])\, u[n-m]$$

$$\sum_{m=5}^{n} u[m-5]\, u[n-m] = n-5+1$$

$n \geq 9$

$$4+\sum_{m=9}^{n} (0{,}5)^{m-8}\, u[m-9] \cdot u[-n+m]$$

$$4+\sum_{m=9}^{n} (0{,}5)^{m-8}\, u[-n+m]$$

$$4+\frac{(0{,}5)^{-8}\,(0{,}5)^{n+1}}{1-0{,}5} = 5-\left(\frac{1}{2}\right)^{n-8}$$

b) $\left(\frac{1}{2}\right)^{|n|} * u[-n+5]$

$n-5<0$

$$\sum_{m=n-5}^{0} \left(\frac{1}{2}\right)^{|m|} = \sum_{m=0}^{-n+5} \left(\frac{1}{2}\right)^{m} = \frac{\left(\frac{1}{2}\right)^{0} - \left(\frac{1}{2}\right)^{-n+5}}{\frac{1}{2}} = 2-2^{n}\cdot 2^{-5} = \underbrace{2-2^{n-5}}_{a}$$

$$\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^{m} = \frac{\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^{\infty}}{\frac{1}{2}} = \underbrace{1}_{b} \rightarrow a+b = 3-2^{n-5}$$

3.8-3)

a) $\left[-1,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i,\ \frac{-1}{2}+\frac{\sqrt{3}}{2}i,\ \frac{1}{2}-\frac{\sqrt{3}}{2}i,\ \frac{1}{2}+\frac{\sqrt{3}}{2}i,\ 1\right]$

b) $x[n] = [\underset{\uparrow\ -4}{16}, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}]$ $\quad x[n-m] = [\underset{\uparrow\ n-3}{\frac{1}{8}}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \underset{\uparrow\ n+4}{16}]$

$h[m] = [2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 2 \ldots$

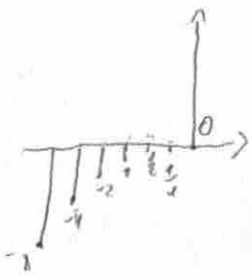
$x[10-m]$ $\quad [\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16]$

$y[n] = 2\left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right) + 2\cdot 4 + 2\cdot 8 + 2\cdot 16 = 57{,}75$

3.8-4)

a)

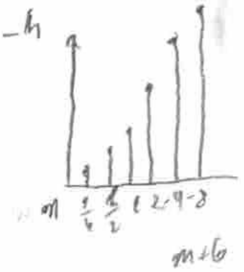
b[n]

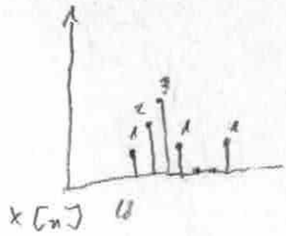


$h[n] \neq 0 \mid n < 0$; system is not causal

b)

$h[m-n]$





$$y[12] = -\left(\frac{1}{4}\cdot 2 + \frac{1}{2}\cdot 3 + 1 + 0 + 0 + 8\right) = -11$$

3.8-5)

a)

$$h[n] = (-2)^n u[n-1]$$

$$x[n] = e^{-n} u[n+1]$$

$$y[n] = \sum_{m=-\infty}^{\infty} (-2)^m u[m-1]\, e^{-(n-m)}\, u[n-m+1]$$

$$y[n] = e^{-n} \sum_{m=-\infty}^{\infty} (-2)^m e^m \underbrace{u[m-1]}_{m \geq 1} \cdot \underbrace{u[n-m+1]}_{m \leq n+1}$$

$$= e^{-n} \sum_{m=1}^{n+1} (-2e)^m = e^{-n} \left[\frac{-2e - (-2e)^{n+2}}{1-(-2e)} \right]$$

$$\frac{e^{-n}\left[-2e - (-1)^{n+2} (e)^{n+2}\right]}{1-(-2e)}$$

$$\frac{-2e^2}{1+2e}\left[e^{-n-1} - (-2)^{n+1}\right] \leftarrow -2e^2\left[\frac{e^{-n-1} - (-2)^{n+1}}{1-(-2e)}\right] \leftarrow \frac{-2e^{-n+1} - (-2)^{n+2} e^2}{1-(-2e)}$$

b)

$$h[n] = (-2)^n u[n-1] = (-2)^{n+1} u[n] = -2[(-2)^n u[n]]$$

$$x[n] = e^{-n} u[n+1] = e^{-(n-1)} u[n] = e[(e^{-n}) u[n]]$$

$$h[n] * x[n] = -2e\left[-2^n u[n] * e^{-n} u[n]\right]$$

Table 3.1 -> 4

$$= -2e\left[\frac{-2^{n+1} - e^{-(n+1)}}{-2-e^{-1}}\right]$$

$$\frac{-2e}{-2-e^{-1}} = \frac{2e}{2+e^{-1}} = \frac{2e^2}{2e+1}$$

$$\Rightarrow y[n] = \frac{2e^2}{2e+1}\left[-2^{n+1} - e^{-(n+1)}\right]$$

3.8-6)

$$x[n] = 3^{n-1} u[n+2] \Rightarrow x[n-2] = 3^{n-3} u[n]$$

$$h[n] = \frac{1}{2}[\delta[n-2] - (-2)^{n+1}] u[n-3] \to h[n+3] = \frac{1}{2}[\delta[n+1] - (-2)^{n+4}] u[n]$$

$$= \frac{1}{2}\delta[n+1] u[n] - (-2)^4 (-2)^n u[n]$$

$$y[n-2+3] = x[n-2] * h[n+3] = \frac{3^{-3}}{2}[3^n \cancel{u[n]} * \delta[n+1]] - \frac{(-2)^4}{2 \cdot 3^3}\left[3^n u[n] * (-2)^n u[n]\right]$$

$$= \frac{-8}{27}\left[\frac{3^{n+1} - (-2)^{n+1}}{3-(-2)}\right]$$

$$y[n+1] = \frac{-8}{135}\left[3^{n+1} - (-2)^{n+1}\right] u[n]$$

$$y[n] = \frac{-8}{135}\left(3^n - (-2)^n\right) u[n-1]$$

3.8-f)

$$x[n] = 3^{n+2} u[n+1] \Rightarrow x[n-1] = 3 \cdot 3^{n} u[n]$$

$$h[n] = [2^{n-2} + 3(-5)^{n+2}] u[n-1] \Rightarrow h[n+1] = [2^{n-1} + 3(-5)^{n+3}] u[n]$$

$$y[n+1-1] = 3[3^{n} * 2^{n-2}] u[n] + 9[3^{n} * (-5)^{n+3}] u[n]$$

$$= \frac{3}{2^2}\left[\frac{3^{n+1} + 2^{n+1}}{3-2}\right] + 9(-5)^3 \left[\frac{3^{n+1} - (-5)^{n+1}}{3+5}\right]$$

$$= \frac{3}{4}[3^{n+1} - 2^{n+1}] + \frac{1125}{8}[3^{n+1} - (-5)^{n+1}]$$

3.8-P)

$$x[n] = 3^{-n+2} u[n+3] \Rightarrow x[n-3] = 3^{-(n-3)+2} u[n] = 3^{5} 3^{-n} u[n]$$

$$h[n] = 3(n-2) 2^{n-3} u[n-4] \Rightarrow h[n+4] = 3(n+2) 2^{n+1} u[n] = 3 \cdot 4 \cdot 2^{n} + 6 \cdot n \cdot 2^{n}$$

$$y[n-3+4] = 3^{5} \cdot 3 \cdot 4 \left(3^{-n} u[n] * 2^{n} u[n]\right) + 3^{5} \cdot 6 \left(3^{-n} u[n] * n 2^{n} u[n]\right)$$

$$y[n+1] = 3^{6} \cdot 4 \left(\frac{3^{-(n+1)} - 2^{n+1}}{3^{-1} - 2}\right) + 3^{5} \cdot 6 \left(\frac{3^{-1} \cdot 2}{(2-3^{-1})^2}\left[2^{n} - 3^{-n} + \frac{2-3^{-1}}{3^{-1}} n 2^{n}\right]\right)$$

$$y[n+1] = \left[\frac{3^{6} \cdot 4}{3^{-1} - 2}\left(3^{-n-1} - 2^{n+1}\right) + \frac{3^{4} \cdot 12}{(2-3^{-1})^2}\left(2^{n} - 3^{-n} + \frac{2-3^{-1}}{3^{-1}} n 2^{n}\right)\right] u[n]$$

$$x[n] = 2^{n} u[n-1] \Rightarrow x[n+1] = 2 \cdot 2^{n} u[n]$$

$$h[n] = 3^{n} \cos\left(\frac{\pi}{3} n - 0{,}5\right) u[n]$$

$$y[n+1] = 2 \cdot \left[x[n+1] * h[n] \right]$$

$$R = \left[3^2 + 2^2 - 2 \cdot 3 \cdot 2 \cos\left(\frac{\pi}{3}\right) \right]^{1/2} = 7^{\frac{1}{2}}$$

$$\phi = \tan^{-1}\left[\frac{3 \sin\left(\frac{\pi}{3}\right)}{3 \cos\left(\frac{\pi}{3}\right) - 2} \right] = -1{,}38$$

$$y[n+1] = \frac{2 \cdot 1}{\sqrt{7}} \left[3^{n+1} \cos\left(\frac{\pi}{3}(n+1) - 0{,}5 + 1{,}38\right) - 2^{n+1} \cos(-0{,}5 + 1{,}38) \right] u[n]$$

$$y[n] = \frac{2}{\sqrt{7}} \left[3^{n} \cos\left(\frac{\pi}{3} n + 0{,}88\right) + 0{,}636 \cdot 2^{n} \right] u[n-1]$$

3.8-10)

a) $(E - \frac{1}{2})\, y[n] = x[n]$

$\lambda = [\frac{1}{2}]$

$$h[n] = \frac{1}{-\frac{1}{2}} \delta[n] + c\left(\frac{1}{2}\right)^{n}$$

$h[0] = x[-1] + \frac{1}{2} h[-1]$

$h[0] = 0$

$-2 + c = 0$

$c = 2$

$$h[n] = -2\,\delta[n] + 2\left(\frac{1}{2}\right)^{n} u[n]$$

$u[n]$

b) $y[n] = h[n] * u[n]$

0 for $m \neq 0$ 0 for $n < m$ 0 for $m < 0$ 0 for $m > n$

$$= -2 \sum_{m=-\infty}^{\infty} \delta[m]\, u[n-m] + 2 \sum_{m=-\infty}^{\infty} \left(\frac{1}{2}\right)^{m} u[m]\, u[n-m]$$

$$= -2\,\delta[0] + 2 \sum_{m=0}^{n} \left(\frac{1}{2}\right)^{m} u[m] = -2\,\delta[0] + 2 \left[\frac{\left(\frac{1}{2}\right)^{0} - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}} \right]$$

$$y[n] = -2\,\delta[0] + 4\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u[n]$$

3.8-11)

1- $\delta[n-k] * x[n] = \sum_{m=-\infty}^{\infty} \underline{\delta[m-k]} \, x[n-m]$

$\hookrightarrow 0$ for $m-k \neq 0$

then for $m=k$

$\delta[0] x[n-k] = x[n-k]$

2- $\gamma^n u[n] * u[n] = \sum_{m=-\infty}^{\infty} \gamma^m \underbrace{u[m]}_{0 \,\forall m<0} \underbrace{u[n-m]}_{0 \,\forall m>n} = \sum_{m=0}^{n} \gamma^m = \left[\frac{\gamma^0 - \gamma^{n+1}}{1-\gamma}\right] u[n]$

$= \left[\frac{1-\gamma^{n+1}}{1-\gamma}\right] u[n]$

3- $u[n] * u[n] = \sum_{m=-\infty}^{\infty} \underbrace{u[m]}_{0 \,\forall m<0} \underbrace{u[n-m]}_{0 \,\forall m>n} = \sum_{m=0}^{n} u[n] = (n+1) u[n]$

3.8-12)

4- $\gamma_1^n u[n] * \gamma_2^n u[n] = \sum_{m=-\infty}^{\infty} \gamma_1^m \underbrace{u[m]}_{0 \,\forall m<0} \gamma_2^{n-m} \underbrace{u[n-m]}_{0 \,\forall m>n} = \gamma_2^n \sum_{m=0}^{n} \gamma_1^m \gamma_2^{-m} u[n]$

$= \gamma_2^n u[n] \sum_{m=0}^{n} \left(\frac{\gamma_1}{\gamma_2}\right)^m = \gamma_2^n u[n] \cdot \left[\frac{\left(\frac{\gamma_1}{\gamma_2}\right)^0 - \left(\frac{\gamma_1}{\gamma_2}\right)^{n+1}}{1-\left(\frac{\gamma_1}{\gamma_2}\right)}\right]$

$= \frac{\gamma_2^n - \frac{1}{\gamma_2}(\gamma_1)^{n+1}}{\frac{\gamma_2-\gamma_1}{\gamma_2}} u[n]$

$= \frac{\gamma_2^{n+1} - \gamma_1^{n+1}}{\gamma_2 - \gamma_1}$

5- $u[n] * n\,u[n] = \sum_{m=-\infty}^{\infty} \underbrace{u[n-m]}_{0\,\forall m>n} \, m \underbrace{u[m]}_{0\,\forall m<0} = \sum_{m=0}^{n} m\,u[n] = u[n] \sum_{m=0}^{n} m$

$$= \frac{n(n+1)}{2} u[n]$$

6- $\gamma^n u[n] * n\,u[n] = \sum_{m=-\infty}^{\infty} \gamma^m \underline{u[m]} (n-m) \underline{u[n-m]} = \sum_{m=0}^{n} \gamma^m (n-m)\, u[n] =$

$$\left[n \sum_{m=0}^{n} \gamma^m - \sum_{m=0}^{n} m\gamma^m\right] u[n] = \left[n\left(\frac{1-\gamma^{n+1}}{1-\gamma}\right) - \gamma \frac{1-(n+1)\gamma^n + n\gamma^{n+1}}{(1-\gamma)^2}\right] u[n]$$

$$= \frac{n(1-\gamma)(1-\gamma^{n+1}) - \gamma + (n+1)\gamma^{n+1} - n\gamma^{n+2}}{(1-\gamma)^2} u[n]$$

$$= \frac{n[1 - \cancel{\gamma^{n+1}} - \gamma + \cancel{\gamma^{n+2}} + \cancel{\gamma^{n+1}} - \cancel{\gamma^{n+2}}] - \gamma + \gamma^{n+1}}{(1-\gamma)^2} u[n]$$

$$= \frac{n(1-\gamma) + \gamma(\gamma^n - 1)}{(1-\gamma)^2} u[n]$$

3.8-13)

7 - $n\,u[n] * n\,u[n] = \sum_{m=-\infty}^{\infty} m\,u[m](n-m)\,u[n-m] = \sum_{m=0}^{n} m(n-m)\,u[n]$

$$= n\sum_{m=0}^{n} m\,u[n] - \sum_{m=0}^{n} m^2 u[n] = \left[n \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}\right] u[n]$$

$$= \left[\frac{3n^2(n+1) - n(n+1)(2n+1)}{6}\right] u[n] = \left[\frac{n(n+1)[3n - (2n+1)]}{6}\right] u[n]$$

$$= \left[\frac{n(n+1)(n-1)}{6}\right] u[n]$$

8- $\gamma^n u[n] * \gamma^n u[n] = \sum_{m=-\infty}^{\infty} \gamma^m u[m] \gamma^{n-m} u[n-m]$

$= \sum_{m=0}^{n} \gamma^m \gamma^{n-m} u[n] = \gamma^n u[n] \sum_{m=0}^{n} \gamma^m \gamma^{-m} = (n+1)\gamma^n u[n]$

3. 8-14)

9- $n \gamma_1^n u[n] * \gamma_2^n u[n] = \sum_{m=-\infty}^{\infty} m \gamma_1^m u[m] \gamma_2^{n-m} u[n-m]$

$= \gamma_2^n u[n] \sum_{m=0}^{n} m \gamma_1^m \gamma_2^{-m} = \gamma_2^n u[n] \sum_{m=0}^{n} m \left(\frac{\gamma_1}{\gamma_2}\right)^m$

$= \gamma_2^n u[n] \cdot \left[\frac{\gamma_1}{\gamma_2} \left(\frac{1-(n+1)\left(\frac{\gamma_1}{\gamma_2}\right)^n + n\left(\frac{\gamma_1}{\gamma_2}\right)^{n+1}}{\left(1-\frac{\gamma_1}{\gamma_2}\right)^2} \right) \right]$

$= u[n] \left[\frac{\gamma_1}{\gamma_2} \left(\frac{\gamma_2^n - (n+1)\gamma_1^n + n\frac{1}{\gamma_2}(\gamma_1)^{n+1}}{\left(1-\frac{\gamma_1}{\gamma_2}\right)^2} \right] \right.$

$= u[n] \left[\frac{\gamma_1}{\gamma_2} \frac{\gamma_2^n - (n+1)\gamma_1^n + n\frac{1}{\gamma_2}(\gamma_1)^{n+1}}{\left(\frac{\gamma_2-\gamma_1}{\gamma_2}\right)^2} \right]$

$= u[n] \left[\frac{\gamma_1\gamma_2}{(\gamma_2-\gamma_1)^2} \gamma_2^n - \gamma_1^n - n\gamma_1^n + n\frac{1}{\gamma_2}\gamma_1^{n+1} \right]$

$= u[n] \left[\frac{\gamma_1\gamma_2}{(\gamma_2-\gamma_1)^2} \gamma_2^n - \gamma_1^n + n\gamma_1^n \left(-1 + \frac{\gamma_1}{\gamma_2}\right) \right]$

$= \left[\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2} \gamma_2^n - \gamma_1^n + n\gamma_1^n \left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right) \right] u[n]$

11- $\gamma_1^n u[-(n+1)] * \gamma_2^n u[n] =$

$$= \sum_{m=-\infty}^{\infty} \gamma_1^m \underbrace{u[-(m+1)]}_{m\leq -1} \gamma_2^{n-m} \underbrace{u[n-m]}_{m\leq n}$$

$$\gamma_2^n \left[\sum_{m=-\infty}^{-1} \left(\frac{\gamma_1}{\gamma_2}\right)^m + \sum_{m=-\infty}^{n} \left(\frac{\gamma_1}{\gamma_2}\right)^m \right]$$

$$\gamma_2^n \left[\frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{-\infty} - \left(\frac{\gamma_1}{\gamma_2}\right)^0}{1-\frac{\gamma_1}{\gamma_2}} \right] + \gamma_2^n \left[\frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{-\infty} - \left(\frac{\gamma_1}{\gamma_2}\right)^{n+1}}{1-\frac{\gamma_1}{\gamma_2}} \right]$$

$$\frac{-\gamma_2^{n+1}}{\gamma_2-\gamma_1} + \frac{-\gamma_1^{n+1}}{\gamma_2-\gamma_1} = \frac{\gamma_2^{n+1}}{\gamma_1-\gamma_2} u[-(n+1)] + \frac{\gamma_1}{\gamma_1-\gamma_2} u[n]$$

3.8-15)

$(E+2)\,y[n] = E\,x[n]$

$\lambda = -2$

$h[0] - 2h[-1] = \delta[0]$

$h[0] = 1$

$h[n] = c(-2)^n$

$h[n] = (-2)^n u[n]$

$y_0[n] = C_0(-2)^n$

$y_0[0] - 2\cdot 10 = 0$

$y_0[0] = 20 \to y_0[n] = 20(-2)^n$

$y[n] = h[n] * x[n]$

$$y[n] = \sum_{m=-\infty}^{\infty} (-2)^m u[m]\, e^{-(n-m)} u[n-m] = \sum_{m=0}^{n} (-2)^m e^{-n} e^m u[n]$$

$$y[n] = e^{-n} u[n] \sum_{m=0}^{n} (-2e)^m = e^{-n} u[n] \left(\frac{1-(-2e)^{n+1}}{1+2e} \right)$$

$$y[n] = \frac{e^{-n} - (-2)^{n+1} e}{1+2e} u[n] \to y_{total} = y[n] + y_0[n]$$

3.8-16)

a) $y[n] = (0,5)^n u[n] * 2^n u[n]$

$$= \left[\frac{(0,5)^{n+1} - 2^{n+1}}{0,5 - 2}\right] u[n] = \left[\frac{2^{n+1} - (0,5)^{n+1}}{1,5}\right] u[n]$$

b) $y[n] = (0,5)^n u[n] * 2^{-3} 2^n u[n]$

$$= \frac{2^{-3}}{1,5} \left(2^{n+1} - 0,5^{n+1}\right) u[n]$$

c) $y[n] = (0,5)^n u[n]\, 2^n u[n-2]$

$$y[n+2] = \frac{2^2}{1,5}\left[2^{n+1} - 0,5^{n+1}\right] u[n]$$

$$y[n] = \frac{2^2}{1,5}\left[2^{n-1} - 0,5^{n-1}\right] u[n-2]$$

3.8-17)

$$y[n] = x[n] - 2x[n-1]$$

Order 1 system. Non recursive, as $y[n]$ does not depend on $y[n-k]$ for any k.
Since the system is non recursive, initial conditions are not needed.

3.8-18)

a) $h[n] = (h_1[n] * h_2[n]) + \left[(h_1[n] * h_5[n] + h_4[n]) * h_3[n]\right]$

b) $h_1[n] = (0,9)^n u[n] - \frac{0,5}{0,9}(0,9)^n (u[n] - \delta[n])$ $\quad h_2[n] = (0,5)^n u[n] - \frac{0,9}{0,5}(0,5)^n (u[n] - \delta[n])$

$$= \frac{4}{9}(0,9)^n u[n] + \frac{5}{9}\delta[n] \qquad = -\frac{4}{5}(0,5)^n u[n] + \frac{9}{5}\delta[n]$$

$$h_1[n] * h_2[n] = -\frac{4}{9}\cdot\frac{4}{5}\left((0,9)^n * (0,5)^n\right) + \frac{9}{5}\frac{4}{9}(0,9)^n u[n] - \frac{4}{5}\frac{5}{9}(0,5)^n u[n] + \frac{5}{9}\frac{9}{5}\delta[n]$$

$$= \delta[n]$$

3.8-19)

a) $g[n] = \sum_{k=-\infty}^{n} h[k]$

for a causal system $h[a]=0$ for $a<0$; thus $h[a]$ is non zero for

$0 \le a \le n$

$$g[n] = \sum_{a=0}^{n} h[a] = \sum_{a=n}^{0} h[n-a] = \sum_{a=0}^{n} h[n-a]$$

b) For a non causal system $h[a] \neq 0$ for some $a<0$ thus

$$g[n] = \sum_{a=0}^{\infty} h[n-a]$$

3.8-20) $h[n] = 2(u[n+2] - u[n-3])$

a) $h[n] = 2(\delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2])$

$y[n] = 2(x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2])$

b) $y[n] = 2^n u[-n] * (2(\delta[n+2] \dots + \delta[n-2])$

$y[n] = 2\sum_{k=-2}^{2} 2^{(n-k)} = 2^{n+4} - 2^{n-1}$

3.8-21)

a) $h_1[n] = [\overset{\downarrow}{2}, -3, 4]$; $h_2[n] = [0, \overset{\downarrow}{0}, -6, -9, 3]$; $h_3 = [\overset{\downarrow}{1}]$

$h_2 \parallel h_3 = [1, 0, -6, -9, 3]$

$h_1 * (h_2 \parallel h_3) = [2, -3, -8, 0, 9, -45, 12]$

b) $x[n] = u[-m]$ $\quad x[m] * h[n-m]$ $\quad h_2 = [\overset{\overset{n-4}{\downarrow}}{3}, -9, -6, 0, 0]$

$n-4 \le -2 \Rightarrow n \le 2 \rightarrow 12$

$n-4 = -1 \rightarrow n=3 \rightarrow -6$

$n-4=0 \rightarrow n=4 \rightarrow 3$

$n>4 \rightarrow 0$

3.8-22) $y[n] = x[n] + (1+M)\,y[n-1]$ for $n<4$

$$y[n] = x[n] + 1{,}01\,y[n-1]$$

$$x[n] = 500\,u[n] - 1500\,\delta[n-4]$$

$$y[n] - 1{,}01\,y[n-1] = x[n]$$

$$\lambda = [1{,}01]$$

$$h[n] = C\,(1{,}01)^n$$

$$h[0] = \delta[0] = 1 \Rightarrow C = 1$$

$$h[n] = (1{,}01)^n$$

$$y[n] = h[n] * x[n]$$

$$= (1{,}01)^n * (500\,u[n] - 1500\,\delta[n-4])$$

$$= 500\left[\frac{1-(1{,}01)^{n+1}}{-0{,}01}\right] - 1500\,(1{,}01)^{n-4}\,u[n-4]$$

3.8-23) $y[n] = (1+M)\,y[n-1] + x[n]$

$$h[n] = (1+M)^n$$

$x[n] = P$ for $n > 0$

$$x[0] = -(M+P)$$

$$x[n] = P\,u[n]$$

$$y[n] = x[n] * h[n] = P\left(\frac{1-(1+M)^{n+1}}{-M}\right) - (M+P)\,(1+M)^n$$

$$= P\left(\frac{1-(1+M)^{n+1}}{-M} - (1+M)^n\right) - M\,(1+M)^n$$

$$= P\left(\frac{1-(1+M)^{n+1} + M(1+M)^n}{-M}\right) - M\,(1+M)^n$$

$$= P\left(\frac{1+(1+M)^n\,(-(1+M)+M)}{-M}\right) - M\,(1+M)^n$$

$$= P\left(\frac{1-(1+M)^n}{-M}\right) - M\,(1+M)^n = P\left(\frac{(1+M)^n - 1}{M}\right) - M\,(1+M)^n$$

$$y[N] = 0 \Rightarrow P\left(\frac{(1+M)^N - 1}{M}\right) = M\,(1+M)^N \Rightarrow P = \frac{M\,(1+M)^N\,M}{(1+M)^N - 1} = \frac{M\,M}{1-(1+M)^{-N}}$$

3+8=24)

$$500 = \frac{1000 \cdot (0{,}015)}{1-(1{,}015)^{-N}}$$

$$500\left(1-(1{,}015)^{-N}\right) = 1000 \cdot (0{,}015)$$

$$1-(1{,}015)^{-N} = \frac{1000 \cdot (0{,}015)}{500}$$

$$-(1{,}015)^{-N} = \frac{1000 \cdot (0{,}015)}{500} - 1$$

$$(1{,}015)^{-N} = 0{,}7$$

$$(1{,}015)^{N} = \frac{1}{0{,}7}$$

$$N \ln(1{,}015) = \ln\left(\frac{1}{0{,}7}\right) \Rightarrow N = \frac{\ln\left(\frac{1}{0{,}7}\right)}{\ln(1{,}015)} = 23{,}95$$

≈ 24 payments

3.8-25)

a) $y_a = [-20, -30, 10, 15, 10, 15]$

b) $y_b = [-2, -7, 3, -15, 13, -8, 4]$

c) $y_c = [0, 6, 6, 13, 2, 6, 12, 6, -4, 3]$

d) $y_d = [-5, 5, 15, 17, -20, 17, 18, 28, -22, 24]$

e) $a = [1, -1] * [1, -1] = [1, 2, 1]$

$y_e = a * a = [1, -4, 6, -4, 1]$

f) $a = [2, -1] * [1, -2] = [2, -5, 2]$

$y_f = a * a = [4, -20, 33, -20, 4]$

3.8-26) $y[n] = (2x[n-30] * (-\frac{3}{2} h[n-10])$

$n=0$

let $g[n] = -3\left[x[n] * h[n]\right] = [+6, +15, +21, +21, +21, -15, -39, -30]$

$n=40$

$y[n] = g[n-30-10] = [+6, +15, +21, +21, +21, -15, -39, -30]$

3.8-32) $y = Hx$

a) $h[n] = \frac{8}{n+1}$

b) $x = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 7/3 \\ 43/9 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/3 \\ 1/9 \end{bmatrix}$

3.8-33)

a) $y_0[n] = \sum_{k=0}^{\lambda} (2+(\frac{1}{3})^k)\,\underset{\substack{\downarrow \\ 0\ \forall\, n \neq k}}{\delta[n-k]} = 2+(\frac{1}{3})^n$

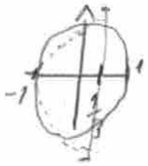
$\hookrightarrow \lambda = [1, 1/3] \rightarrow 2(1)^n + 1(\frac{1}{3})^n$

$(\lambda-1)(\lambda-\frac{1}{3}) = \lambda^2 - \frac{1}{3}\lambda - \lambda + \frac{1}{3} = \lambda^2 - \frac{4}{3}\lambda + \frac{1}{3}$

b) $x[n] = C_1 + C_2(\frac{1}{3})^n$ would produce a strong response as it is composed of the systems natural modes

c) 

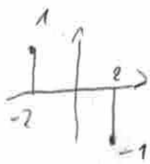
farthest point from $\lambda_1 = -1$

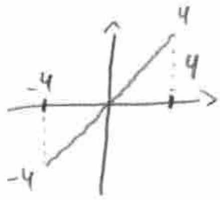
for λ_2 

thus -1 is also a choice

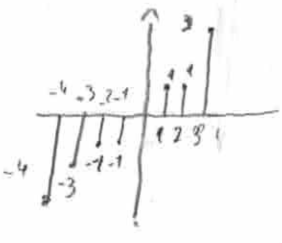
thus $x[n] = (-1)^n u[n]$

3.8-34)

a) $h_1[n]$ 

$h_2[n] =$ 

b) $h_p = h_1[n] + h_2[n]$

$=$ 

c) $h_p = h_1[n] * h_2[n] = [-4, -3, -2, -1, 4, 4, \overset{\substack{n=0 \\ \downarrow}}{4}, 4, 0, -1, -2, -3]$

$=$ 

3.8-35)

a) $(z^3+z^2+z+1)(z^3+z^2+z+1) = z^6+z^5+z^4+z^3+z^5+z^4+z^3+z^2+z^4+z^3+z^2+z+z^3+z^2+z+1$

$= z^6+2z^5+3z^4+4z^3+3z^2+2z+1$

$[1,1,1,1]*[1,1,1,1] = [1,2,3,4,3,2,1]$

c) $[1,0,-2,3]*[1,0,-2,3]*[1,0,-2,3]*[1,0,-2,3]$

$= [1,0,-10,15,40,-120,10,360,-460,-810,1048,-840,-315,1080,-810,243]$

d) $[0,0,0,0,5,0,3,2,1]*[0,0,0,0,5,0,3,2,1]*[1,0,-5,0,13,0,0,0,0]$

$= [25,0,-95,0,204,10,257,-44,321,104,137,58,47,52,13]$

z^{-4} z^0 z^{10}

3.8-36)

a) $y[n] - \frac{1}{3}y[n-1] = x[n]$

b) $x[n] = 2u[n]$

c) $(E-\frac{1}{3})y[n] = E(x[n])$

$\lambda = \frac{1}{3}$

$h[n] = c\left(\frac{1}{3}\right)^n$

$h[0] = \delta[0] = 1 = c$

$h[n] = \left(\frac{1}{3}\right)^n$

$x[n] * h[n]$

$2u[n] * \left(\frac{1}{3}\right)^n u[n]$

$= 2 \cdot \frac{1-3^{-(n+1)}}{1-3^{-1}} u[n]$

$\lim_{n\to\infty} = 2 \cdot \frac{1-3^{-(n+1)}}{1-3^{-1}} = 2 \cdot \frac{1}{1-\frac{1}{3}} = 2 \cdot \frac{1}{\frac{2}{3}} = 3$

3.8-37)

a) $(jE + 0{,}5)\, y[n] = (-5E)\, x[n]$

$\lambda = 0{,}5j$

$h[n] = c\,(0{,}5j)^n$

$j\,h[0] + 0{,}5\,h[-1] = -5\,\delta[0]$

$h[0] = \dfrac{-5}{j} = 5j$

$c = 5j \qquad h[n] = 5j\,(0{,}5j)^n$

b) $y_0[-1] = j = c\,(0{,}5j)^n \Rightarrow \dfrac{c}{0{,}5j} = j \Rightarrow c = -0{,}5$

$y_0[n] = -0{,}5\,(0{,}5j)^n$

$y[n] = x[n] * h[n]$

$= (u[n] - \delta[n-4] - \delta[n-3] - \delta[n-2] - \delta[n-1] - \delta[0]) * h[n]$

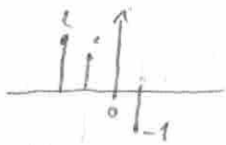
$= h[n] * u[n] - \sum_{n=0}^{4} 5j\,(0{,}5j)^n$

$= 5j\left[\dfrac{1-(0{,}5j)^{n+1}}{1-0{,}5j}\right] - \sum_{n=0}^{4} 5j\,(0{,}5j)^n = 5j\left[\dfrac{1-(0{,}5j)^{n+1}}{1-0{,}5j} - \dfrac{1-(0{,}5j)^5}{1-(0{,}5j)}\right]$

$= 5j\left[\dfrac{1-(0{,}5j)^{n+1} - 1 + (0{,}5j)^5}{1-(0{,}5j)}\right] = 5j\left[\dfrac{(0{,}5j)^5 - (0{,}5j)^{n+1}}{1-(0{,}5j)}\right] u[n]$

$y_{total} = y_0[n] + y[n] = -0{,}5\,(0{,}5j)^n + 5j\left[\dfrac{(0{,}5j)^5 - (0{,}5j)^{n+1}}{1-(0{,}5j)}\right] u[n]$

3.8-38) $h[n] = n(u[n-2] - u[n+2])$

a) 

b) $y[n] = 2x[n+2] + x[n+1] - x[n-1]$

3.8-39)

a) let $z[n]$ have size 1; $x[n] \to 3$, and $y[n] \to 2$

$x[n](z[n] * y[n])$ has size 2 at most

$(x[n] * y[n]) \cdot z[n]$ has size 0 at most

b) let $x[n] \neq 0$ for some $n >$ size $y[n]$ and $z[n]$ but $<$ than the size of $y[n] * z[n]$

$x[n](y[n] * z[n]) \neq 0$ for some n in this case

but $(x[n]y[n]) * (x[n]z[n]) = 0 \;\forall n$

c) The same logic as (b) applies

3.8-40) $y[n] - n\,y[n-1] = x[n]$

a) $h[n] = \delta[n] + n\,h[n-1]$

$h[0] = 1$; $h[1] = 1$; $h[2] = 2$; $h[3] = 6$; $h[4] = 24$, $h[5] = 120$

Not stable, an impulse will lead to infinite output

b) $y[0] = 1$; $y[1] = 2$, $y[2] = 5$, $y[3] = 1 + 3 \cdot 5 = 16$, $y[4] = 1 + 4 \cdot 16 = 65$

c) $x[n] * h[n] = \sum_{n=0}^{n} h[n]$

$$y_c[4] = \sum_{n=0}^{4} h[n] = 34$$

d) shift $x[n] = u[n]$ by 2 such that $x[n] = u[n-2]$

$y[0] = 0$ $\quad y[3] = 4$ //

$y[1] = 0$ $\quad$ thus the system is not time invariant

$y[2] = 1$