3.9-1) let $h[n] = \begin{cases} -1 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$ then $\sum_{n=-\infty}^{\infty} |h[n]| = \infty$

Assume the bounded input $x[k-n] = \begin{cases} 1; & h[n] > 0 \\ -1; & h[n] < 0 \end{cases}$

This $x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$ let $n = k$

$$= \sum_{m=-\infty}^{\infty} |h[m]| = \infty$$

3.9-2)

a) $E^2 + 0{,}6E - 0{,}16 \rightarrow [-0{,}8; 0{,}2] \rightarrow$ Stable in both senses

b) $E^2 + 3E + 2 \rightarrow [-2; -1] \rightarrow$ Unstable in both senses

c) $(E-1)^2(E+\frac{1}{2}) \rightarrow [-\frac{1}{2}; 1; 1] \rightarrow$ Unstable in both senses; repeated roots on UC

d) $E^2 + 2E + 0{,}96 \rightarrow [-1{,}2; -0{,}8] \rightarrow$ Unstable in both senses

e) $E^2 + E - 2E \rightarrow [-2; 1] \rightarrow$ Unstable in both senses

f) $(E^2-1)(E^2+1) = E^4 \cancel{+E^2} \cancel{-E^2} - 1 = [-1, i, -i, 1] \rightarrow$ Marginally stable BIBO unstable

3.9-3)

$h_1[n] = 2^n u[n]$

$h_2[n] = \delta[n] - 2\delta[n-1]$

$$h_c[n] = 2^n u[n] * \delta[n] - 2^n * 2\delta[n-1]$$

$$= 2^n u[n] - 2\, 2^{n-1} u[n-1]$$

$$= 2^n u[n] - 2^n u[n-1]$$

$$= 2^n (u[n] - u[n-1])$$

$$= 2^n (\delta[n])$$

$$\sum_{n=-\infty}^{\infty} |2^n \delta[n]| < \infty$$

The system is stable

3.9-4)

a] D; E; I → outside unit circle

b) A; B; C; E; G; I → real or complex conjugate

c) A; B; C; D; E; F; G; H; I

d) D; F; J; B

e) J; H; F

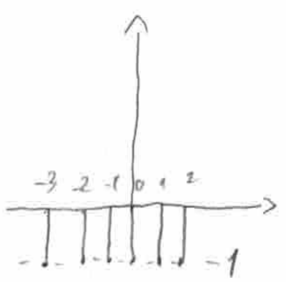
3.9-5)

a) The system is causal as $h[n] = 0 \ \forall n < 0$

$$\sum_{n=0}^{\infty} |h[n]| = 1 + \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = 1 + \frac{\frac{1}{3} - \left(\frac{1}{3}\right)^{\infty}}{1 - \frac{1}{3}} = 1 + \frac{\frac{1}{3}}{\frac{2}{3}} < \infty$$

The system is stable

b)



c) $y[n] = h[n] * x[n] = \left(\frac{1}{3}\right)^n u[n] * u[n-3] - \left(\frac{1}{3}\right)^n u[n] * u[n+3]$

$$= \sum_{m=0}^{n-3} \left(\frac{1}{3}\right)^m - \sum_{m=0}^{n+3} \left(\frac{1}{3}\right)^n = \left[\frac{1 - \left(\frac{1}{3}\right)^{n-2}}{1 - \frac{1}{3}} - \frac{1 - \left(\frac{1}{3}\right)^{n+4}}{1 - \frac{1}{3}} \right]$$

$$= \frac{\left(\frac{1}{3}\right)^{n+4} - \left(\frac{1}{3}\right)^{n-2}}{2/3}$$

$$y[n] = -\frac{364}{27} \left(\frac{1}{3}\right)^n u[n]$$

3.9-6) $h[n] = \left(\frac{1}{2}\right)^{|n|}$

a) No $h[n] \neq 0 \ \forall n < 0$

b) $\sum_{n=-\infty}^{\infty} |h[n]| = 2\sum_{n=0}^{\infty} h[n] - 1 = 2 \frac{1-\left(\frac{1}{2}\right)^{\infty}}{1-\frac{1}{2}} - 1 = 2 \cdot \frac{1}{\frac{1}{2}} - 1 = 3 < \infty$

Stable

c) $E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=5}^{\infty} 9 \to \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=5}^{N} 9 = \lim_{N\to\infty} \frac{(N-5)9}{2N+1}$

$= \lim_{N\to\infty} \frac{9N-45}{2N+1} = \lim_{N\to\infty} \frac{9-\frac{45}{N}}{2+\frac{1}{N}} = \frac{9}{2}$

d)

$y[10] = \sum_{m=-\infty}^{\infty} h[m]\, x[10-m] = \sum_{m=-\infty}^{-1} \left(\frac{1}{2}\right)^{-m} 3\, u[10-5-m] + \sum_{m=0}^{\infty} \left(\frac{1}{2}\right)^{m} u[10-5-m]$

$10-5-m \geq 0$

$-m \geq -10+5$

$m \leq 10-5$

$= 3\sum_{m=-\infty}^{-1} 2^{m} + 3\sum_{m=0}^{5} \left(\frac{1}{2}\right)^{m} = 3 \frac{2^{-\infty}-2^{0}}{1-2} + 3 \frac{1-\left(\frac{1}{2}\right)^{6}}{1-\frac{1}{2}}$

$= 3 \cdot 1 + 3 \frac{63}{32}$

3.10-1)

Root needs to be at $\frac{1}{3}$ $\to$ $(E-\frac{1}{3})y[n] = x[n]$

3.10-2)

$E^2+1 \to \lambda = [i ; -i]$

The resonant input would be of the form $C_1(i)^n + C_2(-i)^n = x[n]$

$$\Rightarrow x[n] = C_1\left(e^{i\frac{\pi}{2}}\right)^n + C_2\left(e^{-i\frac{\pi}{2}}\right)^n \text{ ; let } C_1 = C_2 = C$$

$$= C\left[e^{i\frac{\pi}{2}n} + e^{-i\frac{\pi}{2}n}\right] = 2C\cos\left(\frac{\pi}{2}n\right)$$

3.10-3)

$$h_1[n] = -\left(\frac{1}{2}\right)^n u[n] \Rightarrow \sum_{n=0}^{\infty} -\left(\frac{1}{2}\right)^n = -\frac{1-\left(\frac{1}{2}\right)^\infty}{1-\frac{1}{2}} = -\frac{1}{\frac{1}{2}} = -2 \to \text{Aprox area } 2$$

$h_2[n] = 2(u[n]-u[n-4]) \to$ area $= 4 \cdot 2 = 8$

$Th_1 = \frac{2}{1} = 2$ samples $\qquad$ Th_1 is more eficient

$Th_2 = \frac{8}{2} = 4$ samples